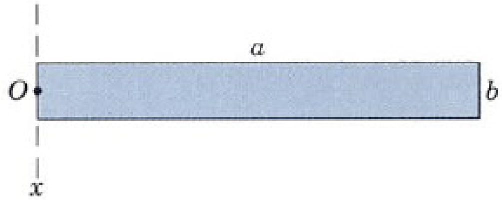
Indian Institute of Technology, Guwahati

ME 101 – Engineering Mechanics

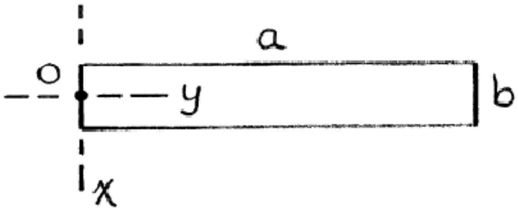
Assignment

---

(1) Considering the Fig. Prob. 01, show that the moment of inertia of the rectangular area about the x-axis through one end may be used for its polar moment of inertia about point O where b is considered small as compared with a. What is the percentage error n where $b/a = 1/10$.



Solu.



Rectangle moment of inertia w.r.t. x, $I_x = \frac{1}{3}Aa^2 = \frac{1}{3}(ab)a^2 = \frac{1}{3}a^3b$

Rectangle moment of inertia w.r.t. y, $I_y = \frac{1}{3}A\left(\frac{b}{2}\right)^2 = \frac{1}{12}ab^3$

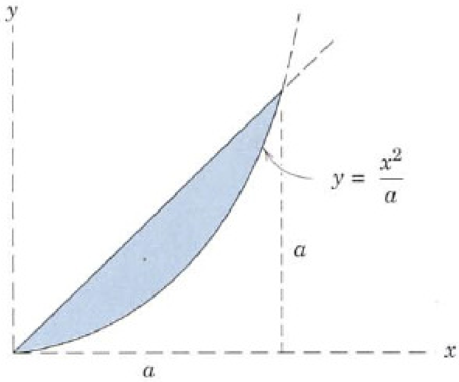
Polar moment of inertia at O, $I_o = I_x + I_y = \frac{1}{3}ab\left(a^2 + \frac{b^2}{4}\right)$

Evaluating error percentage, $n = Error\ \% = \frac{I_x - I_o}{I_o} \times 100 = \frac{-\frac{1}{12}ab^3}{\frac{1}{3}ab\left(a^2+\frac{b^2}{4}\right)} \times 100$

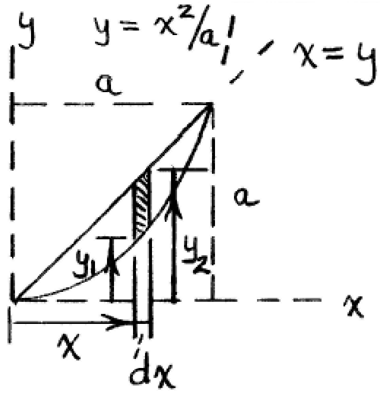
$$\Rightarrow n = -\frac{1}{4}\frac{b^2}{a^2 + \frac{b^2}{4}} \times 100 = -\frac{1}{4}\frac{\left(\frac{b}{a}\right)^2}{1 + \frac{1}{4}\left(\frac{b}{a}\right)^2} \times 100$$

$$\Rightarrow n = -\frac{1}{4}\frac{0.01}{1 + \frac{1}{4} \times 0.01} \times 100 = -0.249\%$$

(2) Determine the moment of inertia of the shaded area (shown in Fig. Prob. 02) about the x- and y-axes. Use the differential element for both the calculations.



Solu.



Differential area, $dA = (y_2 - y_1)dx = \left(x - \frac{x^2}{a}\right) dx$

Rectangle moment of inertia w.r.t. x, $I_x = \int_0^a \frac{1}{3} y_2^3 dx - \int_0^a \frac{1}{3} y_1^3 dx = \frac{1}{3}\int_0^a \left(x^3 - \frac{x^6}{a^3}\right) dx$

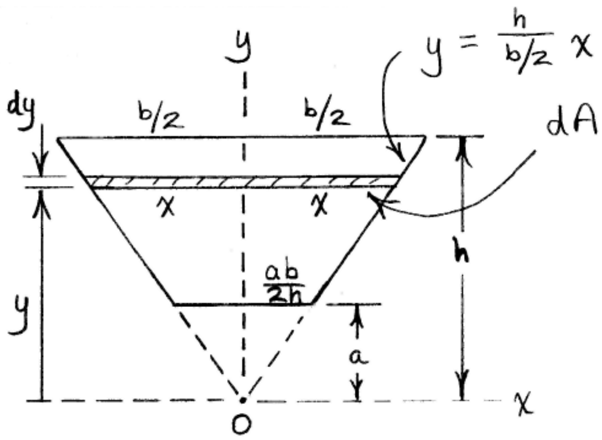
$$\Rightarrow I_x = \frac{1}{3}\left[\frac{x^4}{4} - \frac{x^7}{7a^3}\right]_0^a = \frac{a^4}{3}\left(\frac{1}{4} - \frac{1}{7}\right) = \frac{a^4}{28}$$

Rectangle moment of inertia w.r.t. y, $I_y = \int_0^a x^2 dA = \int_0^a x^2 \left(x - \frac{x^2}{a}\right) dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right) dx$

$$\Rightarrow I_y = \left[\frac{x^4}{4} - \frac{x^5}{5a}\right]_0^a = a^4\left(\frac{1}{4} - \frac{1}{5}\right) = \frac{a^4}{20}$$

(3) Determine the rectangular moments of inertia of the shaded area about the x- and y-axes and the polar radius of gyration about point O.

Solu.



Slant height y in terms of x, $y = \frac{h}{b/2}x = \frac{2h}{b}x$

Differential area, $dA = 2x\,dy = 2\left(\frac{b}{2h}y\right)dy = \frac{b}{h}y\,dy$

Rectangle moment of inertia w.r.t. x, $I_x = \int_a^h y^2 dA = \int_a^h y^2 \frac{b}{h} y\,dy = \frac{b}{h}\left[\frac{y^4}{4}\right]_a^h$

$$\Rightarrow I_x = \frac{b}{4h}(h^4 - a^4)$$

Rectangle moment of inertia w.r.t. y, $I_y = \int_a^h dI_y = \int_a^h \frac{1}{12}(2x)^3\,dy = \frac{2}{3}\int_a^h \left(\frac{b}{2h}y\right)^3 dy$

$$\Rightarrow I_y = \frac{1}{12}\frac{b^3}{h^3}\left[\frac{y^4}{4}\right]_a^h = \frac{1}{48}\frac{b^3}{h^3}(h^4 - a^4)$$

Polar moment of inertia, $I_z = I_x + I_y = \frac{b}{4h}\left(1 + \frac{b^2}{12h^2}\right)(h^4 - a^4)$

Total area, $A = \frac{bh}{2} - \frac{1}{2}\left(\frac{ab}{h}\right)(a) = \frac{b}{2}\left(h - \frac{a^2}{h}\right)$

Radius of gyration, $k_o = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{1}{2}\left(1 + \frac{b^2}{12h^2}\right)(h^2 - a^2)}$

(4) Determine the product of inertia about x-y axes of the circular area with three equal square holes.

Solu.

Shape	Area (A), in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\bar{y}A$, in.^{4}
Circle	$\pi 10^2$	0	0	0
Rectangle 1	$-(3\times 3)$	3.5	-3.5	110.25
Rectangle 2	$-(3\times 3)$	-3.5	-3.5	-110.25
Rectangle 3	$-(3\times 3)$	-3.5	3.5	110.25
			Total, $\sum\bar{x}\bar{y}A$	110.25

(5) The maximum and minimum moments of inertia of the shaded area are 12×10^6 mm^4 and 2×10^6 mm^4, respectively, about axes passing through the centroid C, and the product of inertia with respect to the x-y axes has a magnitude of 4×10^6 mm^4. Use the proper sign for the product of inertia and calculate I_x and the angle α measured counterclockwise from the x-axis to the axis of maximum moment of inertia.

Solu.

From figure we can easily depict that the sign of I_{xy} is negative $(-)$. Also, from the equation of principal moment of inertia, $I_{max}+I_{min}=I_x+I_y$

$I_x+I_y=(12+2)\times 10^6=14\times 10^6$ mm^4

Similarly, $I_x-I_y=\sqrt{\left[2I_{max}-\left(I_x+I_y\right)\right]-4I_{xy}^2}$

$=\sqrt{[2\times 12-14]^2\times 10^{12}-4\times(-4)^2\times 10^{12}}=\sqrt{36\times 10^{12}}=6\times 10^6$ mm^4

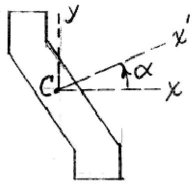
From above two expressions, we get

$I_x=10\times 10^6$ mm^4 and $I_y=4\times 10^6$ mm^4

Principal axes,

$$\tan 2\theta=-\frac{2I_{xy}}{I_x-I_y}=\frac{2\times(-4)\times 10^6}{-6\times 10^6}=\frac{4}{3}$$

$$2\theta = 53.13° \Rightarrow \theta = 26.6°$$



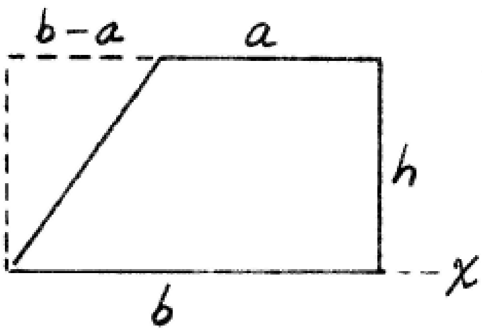
(6) Derive the expression for the moment of inertia of the trapezoidal area about the x-axis through its base.

Solu.

Distort to a rectangle and a triangle without altering y-distribution of area.

Rectangle $I_x = \frac{1}{3}bh^3$ and Triangle $I_x = -\frac{(b-a)h^3}{4}$

For trapezoid, $I_x = \frac{bh^3}{3} - \frac{(b-a)h^3}{4} = \frac{1}{12}(b+3a)h^3$



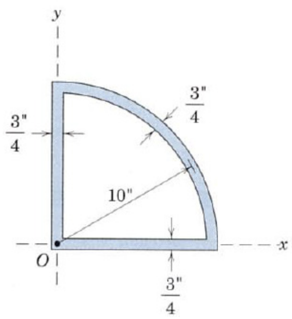
Indian Institute of Technology, Guwahati

ME 101 – Engineering Mechanics

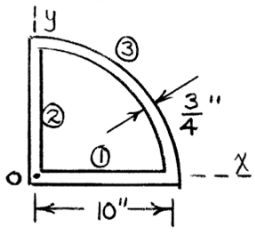
Assignment

---

(7) Calculate the polar radius of gyration about point O of the area shown. Note that the widths of the elements are small compared with their lengths.



Solu.



For Part 1 and Part 2:

Area, $A_1,\ A_2 = 10 \times \frac{3}{4}\ \text{in}^2$

Polar moment of inertia, $I_{o,1}\ ,\ I_{o,2} = \frac{1}{3}\left(10 \times \frac{3}{4}\right) \times 10^2 = \frac{10^3}{4}\ \text{in}^4$

Part 3:

Area, $A_3 = \left\{\frac{\pi(10)}{2} \times \frac{3}{4}\right\}$

Polar moment of inertia, $I_{o,3} = A_3 r^2 = \left\{\frac{3\pi(10)}{8}\right\} 10^2 = \frac{3\pi}{8} \times 10^3\ \text{in}^4$

Combined:

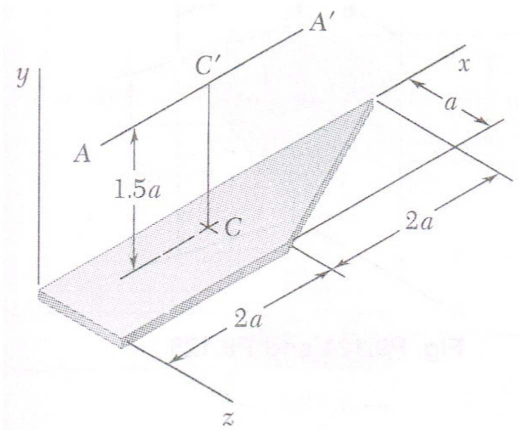
Area, $A = 2 \times \left(10 \times \frac{3}{4}\right) + 10 \times \left(\frac{3\pi}{8}\right) = 26.78\ \text{in}^2$

$I_o = 2 \times \left(\frac{10^3}{4}\right) + \frac{3\pi}{8}(10^3) = 1.68 \times 10^3 \text{in}^4$

Radius of gyration, $k_o = \sqrt{\frac{I_o}{A}} = \sqrt{\frac{1.68\times10^3}{26.78}} = 7.92$ in

(8) A thin plate of mass m has the trapezoidal shape shown in the Fig. Prob. 08. Determine the mass moment of inertia of the plate with respect to (a) the x-axis and (b) the y-axis.

Solu.



(a)

Total area of the trapezoidal shape, $= 2a \times a + \frac{1}{2} \times 2a \times a = 3a^2$

Density of the material (as it has uniform thickness) $\rho = \frac{m}{3a^2}$

Rectangular moment of inertia, $I_x = I_{x,Rect.} + I_{x,Tri.}$

Moment of inertia of the sub-shape rectangle, $I_{x,Rect.} = \frac{1}{3}(2a)(a^3) = \frac{2}{3}a^4$

Moment of inertia of the sub-shape triangle, $I_{x,Tri.} = \frac{1}{12}(2a)(a^3) = \frac{1}{6}a^4$

Thus, the area moment of inertia, $I_x = \frac{2}{3}a^4 + \frac{1}{6}a^4 = \frac{5}{6}a^4$

Mass moment of inertia $= I_x \times \rho$

$$= \frac{5}{6}a^4 \times \frac{m}{3a^2} = \frac{5}{18}ma^2$$

(b)

Polar area moment of inertia, $I_o = I_{o,Rect.} + I_{o,Tri.}$

Rectangle,

$$I_{o,Rect.} = I_{x,Rect.} + I_{z,Rect.} = \frac{2}{3}a^4 + \frac{1}{3}(a)(2a)^3 = \frac{10}{3}a^4$$

Triangle, at the centroid of triangle O'

$$I_{o\prime,Tri.} = I_{x\prime,Tri.} + I_{z\prime,Tri.} = \frac{1}{36}(2a)(a)^3 + \frac{1}{36}(a)(2a)^3 = \frac{5}{18}a^4$$

Distance between the centroid of the triangle sub-shape and origin,

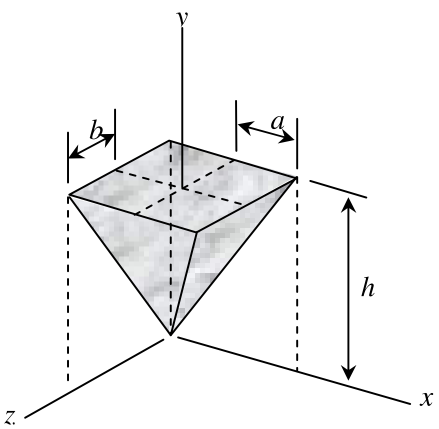
$$d = \sqrt{\left(\frac{a}{3} - 0\right)^2 + \left(2a + \frac{2a}{3} - 0\right)^2} = \sqrt{\frac{65}{9}}a$$

$$I_{o,Tri.} = I_{o\prime,Tri.} + d^2A_{Tri.} = \frac{5}{18}a^4 + \frac{65}{9}a^2(a^2) = \frac{135}{18}a^4 = \frac{15}{2}a^4$$

Total polar area moment of inertia, $I_O = \frac{10}{3}a^4 + \frac{15}{2}a^4 = \frac{65}{6}a^4$

Polar mass moment of inertia $= \frac{65}{6}a^4 \times \frac{m}{3a^2} = \frac{65}{18}ma^2 = 3.61ma^2$

(9) Determine by direct integration of the mass moment of inertia with respect to y-axis of the pyramid shown assuming that it has a uniform density and a mass m.



Solu.

Volume of pyramid, $V = \frac{(2a)(2b)h}{3} = \frac{4abh}{3}$

Density of the material, $\rho = \frac{3m}{4abh}$

Differential mass moment of inertia of the pyramid w.r.t. its centroid axis parallel to y-axis is

$$dI_y = \frac{1}{12}(x^2 + z^2)dm$$

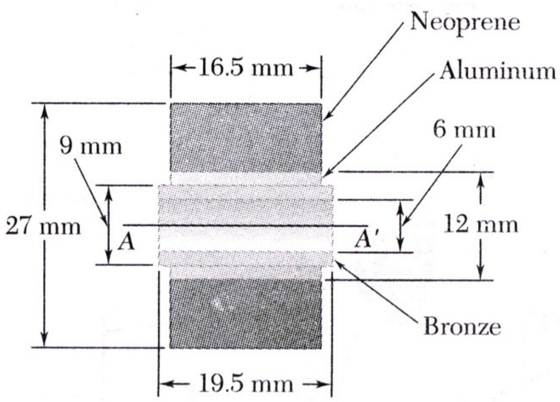
x and z defines the variation of side of any differential rectangle area of thickness dy, both varies w.r.t. y as $2a\left(1-\frac{y}{h}\right)$ and $2b\left(1-\frac{y}{h}\right)$, respectively. Similarly, differential mass $dm = \rho xz\, dy = 4\rho ab\left(1-\frac{y}{h}\right)^2 dy$ .Hence, substituting the values back,

$$\Rightarrow dI_y = \frac{4}{12}(a^2+b^2)\left(1-\frac{y}{h}\right)^4 4\rho ab\, dy$$

$$\Rightarrow \int dI_y = \frac{4\rho ab}{3}(a^2+b^2)\int_0^h \left(1-\frac{y}{h}\right)^4 dy$$

$$\Rightarrow I_y = \frac{\rho abh}{15}(a^2+b^2) = \frac{m}{5}(a^2+b^2)$$

(10) Shown is the cross section of ideal roller. Determine its mass moment of inertia and its radius of gyration with respect to the axis AA'. (The density of bronze is 8580 kg/m^3; of aluminum, 2770 kg/m^3; and of neoprene, 1250 kg/m^3.)



Solu.

Cylinder's mass moment of inertia is $= \frac{1}{2}mr^2$

Mass, $m = \pi r^2 h\rho$

Bronze:

$$I_1 = \frac{1}{2}\left(m_i r_i^2 - m_j r_j^2\right) = \frac{\pi\rho h}{2}\left(r_i^4 - r_j^4\right) = \frac{\pi \times 8580 \times 0.0195}{2}(0.0045^4 - 0.003^4)$$

$$I_1 = 8.648 \times 10^{-8} \text{kg m}^2$$

Mass, $m_1 = \pi h\rho\left(r_i^2 - r_j^2\right) = \pi \times 8580 \times 0.0195(0.0045^2 - 0.003^2) = 5.913 \times 10^{-3}\text{kg}$

Aluminum:

$$I_2 = \frac{\pi \times 2770 \times 0.0165}{2}(0.006^4 - 0.0045^4)$$

$$I_2 = 6.360 \times 10^{-8} \text{kg m}^2$$

Mass, $m_2 = \pi \times 2770 \times 0.0165(0.006^2 - 0.0045^2) = 2.261 \times 10^{-3}\text{kg}$

Neoprene:

$$I_3 = \frac{\pi \times 1250 \times 0.0165}{2}(0.0135^4 - 0.006^4)$$

$$I_3 = 1.034 \times 10^{-6} \text{kg m}^2$$

Mass, $m_3 = \pi \times 1250 \times 0.0165(0.0135^2 - 0.006^2) = 9.476 \times 10^{-3}\text{kg}$

Combined:

$$I = 8.648 \times 10^{-8} + 6.360 \times 10^{-8} + 1.034 \times 10^{-6}$$

$$I = 1.184 \times 10^{-6} \text{kg m}^2$$

Mass, $m = 5.913 \times 10^{-3} + 2.261 \times 10^{-3} + 9.476 \times 10^{-3} = 1.765 \times 10^{-2}\text{kg}$

Radius of gyration, $d = \sqrt{\frac{I}{m}} = \sqrt{\frac{1.184 \times 10^{-6}}{1.765 \times 10^{-2}}} = 8.191 \times 10^{-3}\text{m}.$