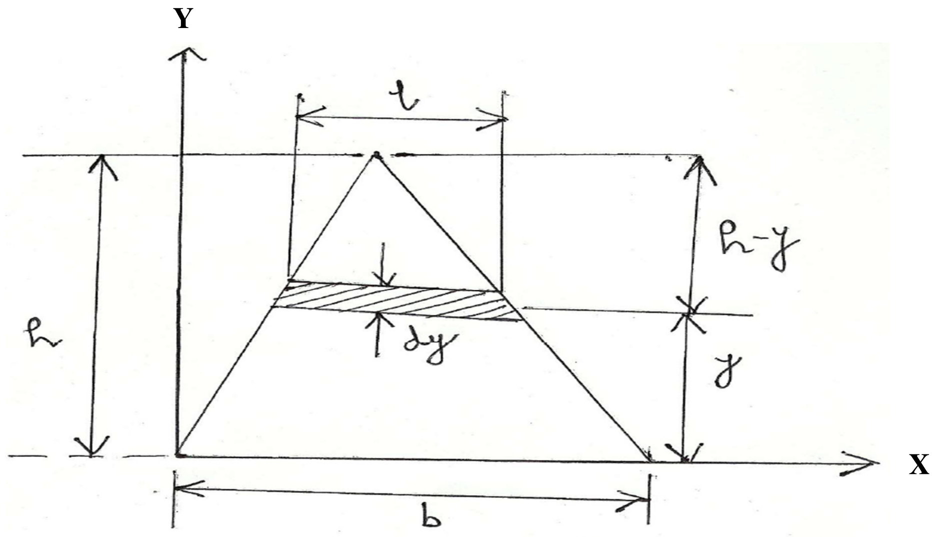
Q. No. 1 Solution



A differential strip parallel to the x axis is chosen for dA

$$dI_x = y^2 dA \qquad dA = l dy$$

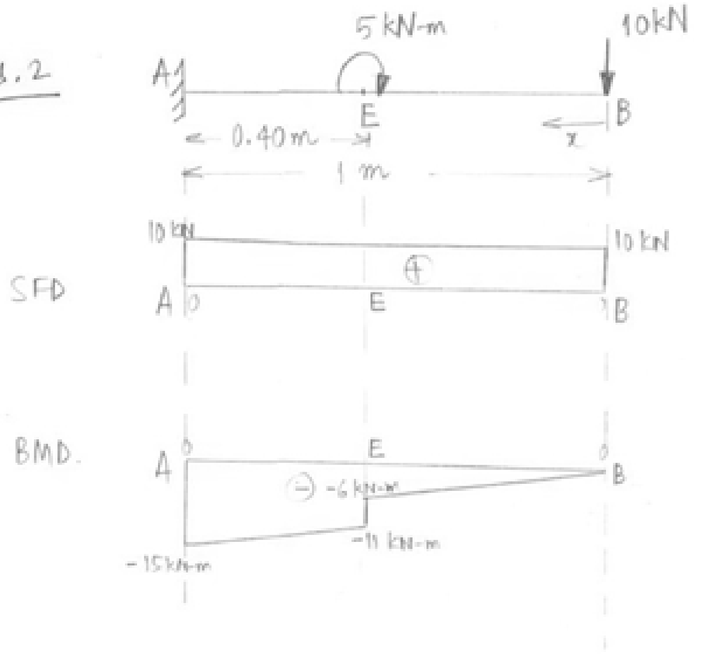
For similar triangles,

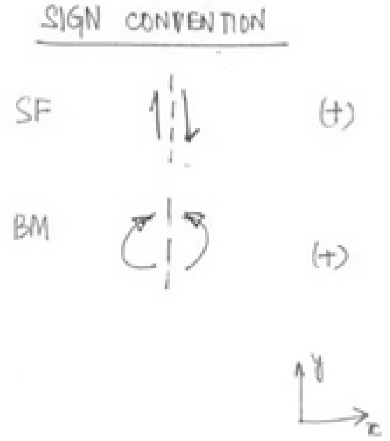
$$\frac{l}{b} = \frac{h-y}{h} \qquad l = b\frac{h-y}{h} \qquad dA = b\frac{h-y}{h}dy$$

Integrating dI_x from $y=0$ to $y=h$,

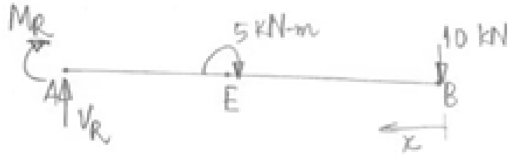
$$I_x = \int y^2\, dA = \int_0^h y^2 b\frac{h-y}{h}dy = \frac{b}{h}\int_0^h (hy^2 - y^3)dy = \frac{b}{h}\left[h\frac{y^3}{3} - \frac{y^4}{4}\right]_0^h \qquad I_x = \frac{bh^3}{12}$$

Ans. 2





Drawing free-body diagram of the beam,



$\Sigma F_y = 0$ (↑+) $\Rightarrow -10 + V_R = 0$

$\Rightarrow V_R = 10 \text{ kN } (\uparrow)$

$\Sigma M_B = 0$ (↺+) $\Rightarrow -M_R - 5 - V_R \times 1 = 0$

$\Rightarrow M_R = -5 - 10$

$\therefore M_R = -15 \text{ kN-m}$ (↺)

For SFD and BMD, a section of beam is isolated and free-body diagram is shown as

→ Section EB,

$\Sigma F_y = 0 \Rightarrow V_E - 10 = 0$

$\Rightarrow \boxed{V_E = 10 \text{ kN}}$

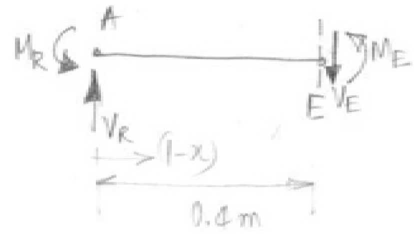
$\Sigma M_B = 0 \Rightarrow -M_E - V_E \times x = 0$

$\Rightarrow \boxed{M_E = -10x \text{ kN-m}}$

Thus, BM at B $(x=0)$ is 0 kN-m and

BM at E $(x = 0.6)$ is -6 kN-m.

→ Now, Section AB



$\sum F_y = 0 \quad \Rightarrow V_R - V_E = 0$

$\Rightarrow \boxed{V_E = 10 \text{ kN}}$

$\sum M_A = 0 \quad \Rightarrow M_R - V_E(1-x) + M_E = 0$

$\Rightarrow M_E = -15 + 10(1-x)$

$\Rightarrow \boxed{M_E = -5 - 10x \text{ kN-m}}$

Thus, BM at E ($x = 0.6$) is -11 kN-m and

BM at A ($x = 1$) is -15 kN-m.

Q. No. 3 (a) Solution

Joint F: $F_{BF} = 0$

Joint B: $F_{BG} = 0$

Joint G: $F_{GJ} = 0$

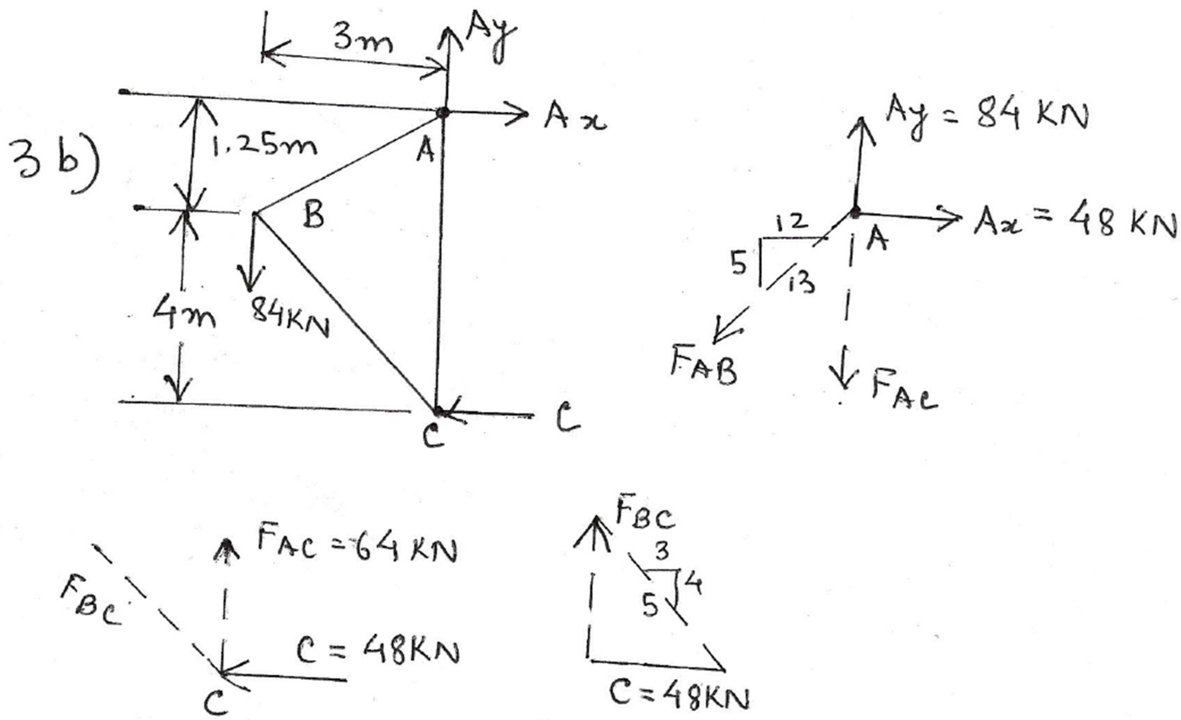
Joint D: $F_{DH} = 0$

Joint J: $F_{HJ} = 0$

Joint H: $F_{EH} = 0$

The zero-force members, therefore, are BF, BG, DH, EH, GJ, HJ

Q. No. 3 (b) Solution



$AB = \sqrt{3^2 + 1.25^2} = 3.25\, m$ $\qquad$ $BC = \sqrt{3^2 + 4^2} = 5\, m$

Reactions

$+\circlearrowleft \quad \sum M_A = 0: \quad (84\, kN)(3\, m) - C(5.25\, m) = 0, \quad C = 48\, kN \leftarrow$

$\overset{+}{\rightarrow} \quad \sum F_x = 0: \; A_x - C = 0, \quad A_x = 48\, kN \rightarrow$

$+\uparrow \quad \sum F_y = 0: \quad A_y = 84\, kN \uparrow$

Joint A:

$\overset{+}{\rightarrow} \quad \sum F_x = 0: \; 48\, kN - \frac{12}{13} F_{AB} = 0, \qquad F_{AB} = +52\, kN \quad T$

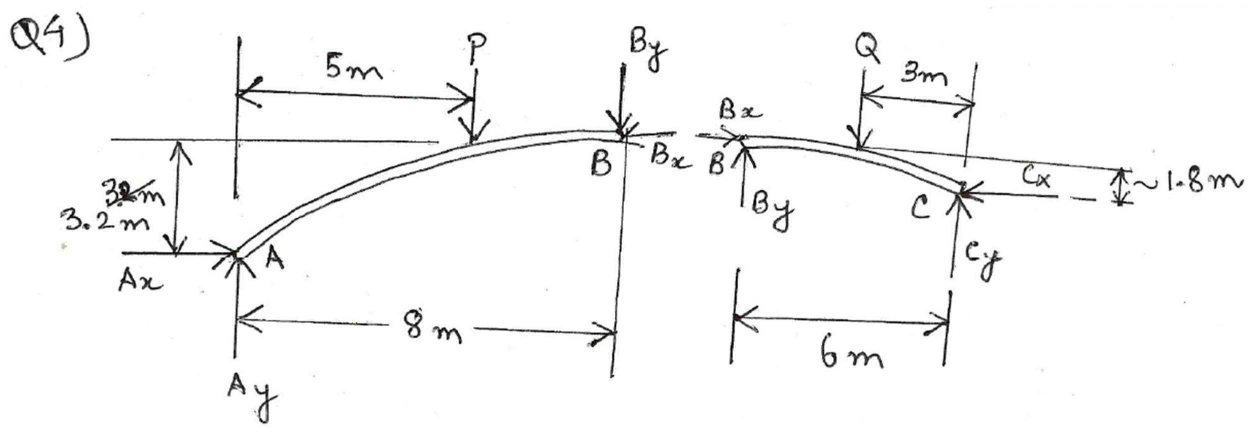
$+\uparrow \quad \sum F_y = 0: \; 84\, kN - \frac{5}{13}(52\, kN) - F_{AC} = 0, \qquad F_{AC} = +64\, kN \quad T$

Joint C:

$$\frac{F_{BC}}{5} = \frac{45\, kN}{3} \qquad F_{BC} = 80\, kN \qquad C$$

Q. No. 4 Solution

4 (a)



Free body: Segment AB

$$+\circlearrowleft \ \sum M_A = 0: \quad B_x(3.2\,m) - B_y(8\,m) - P(5\,m) = 0 \qquad (1)$$

$$0.75\ (Eq\ 1) \quad B_x(2.4\,m) - B_y(6\,m) - P(3.75\,m) = 0 \qquad (2)$$

Free body: Segment BC

$$+\circlearrowright \ \sum M_C = 0: \quad B_x(1.8\,m) + B_y(6\,m) - Q(3\,m) = 0 \qquad (3)$$

$$Add\ (2) and\ (3): \quad 4.2\,B_x - 3.75P - 3Q = 0 \qquad B_x = (3.75P + 3Q)/4.2 \qquad (4)$$

$$Eq\ (1): \quad (3.75P + 3Q)\frac{3.2}{4.2} - 8B_y - 5P = 0 \qquad B_y = (-9P + 9.6Q)/33.6 \qquad (5)$$

Given that $P = 112$ kN and $Q = 140$ kN

4 (b)

Reactions at A:

$$\overset{+}{\rightarrow} \ \sum F_x = 0: \quad A_x - B_x = 0: \quad A_x = B_y = 200\ kN \qquad A_x = 200\ kN \rightarrow$$

$$+\uparrow \ \sum F_y = 0: \ A_y - P - B_y = 0$$

$$A_y - 112\ kN - (\,10\ kN) = 0 \qquad A_y = 122\ \ kN$$

4 (c)

Force exerted at B on AB:

$Eq.(4)$: $B_x = (3.75 \times 112 + 3 \times 140)/4.2 = 200\ kN\ \leftarrow$

$Eq.(5)$: $B_y = (-9 \times 112 + 9.6 \times 140)/33.6 = 10\ kN\ \downarrow$

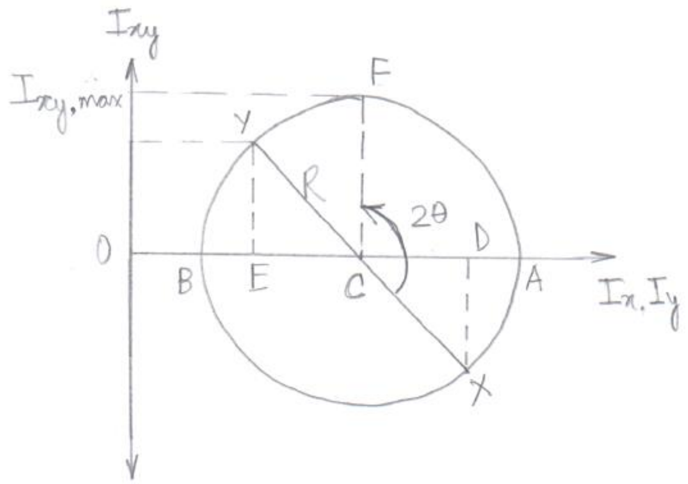
Answer to Q. No 5

given:

$I_y = 1.24869 \times 10^{-4}\ m^4$

$I_{xy} = -5.20289 \times 10^{-5}\ m^4$

Mohr circle is drawn as shown in the figure.



CF corresponds to the maximum product of inertia, i.e $I_{xy,max}$.
Thus, the x axis lies $2 \times 67.5°$ (2θ) clockwise from CF.

$\angle XCF = 2\theta = 135°$.

Hence, the x axis is located as shown.

Considering Δ EYC,

We know that, $\angle YCE = 45°$.

$YE = -|I_{xy}| = 5.20289 \times 10^{-5}\ m^4$

$EC = OC - OE$

$= I_{ave} - I_y$

By trignomotry,

$$\tan(45°) = \frac{YE}{EC} = \frac{-I_{xy}}{I_{ave} - I_y}$$

$$\Rightarrow 1 = \frac{-I_{xy}}{I_{ave} - I_y}$$

$$\Rightarrow I_{ave} = 5.20289 \times 10^{-5} + 1.24869 \times 10^{-4}$$

$$\Rightarrow OC = I_{ave} = 1.7690 \times 10^{-4}\ m^4$$

(a) As,

$$I_{ave} = \frac{I_x + I_y}{2}$$

$$\Rightarrow I_x = 2 I_{ave} - I_y = 2 \times 1.7690 \times 10^{-4} - 1.2487 \times 10^{-4}$$

$$\Rightarrow \boxed{I_x = 2.2893 \times 10^{-4} \; m^4}$$

(b)

$$I_{max, min} = I_{ave} \pm R$$

where $R \rightarrow \sin 45^\circ = \frac{YE}{YC}$

$$\Rightarrow R = \frac{-I_{xy}}{\sin 45^\circ} = 7.3582 \times 10^{-5} \; m^4$$

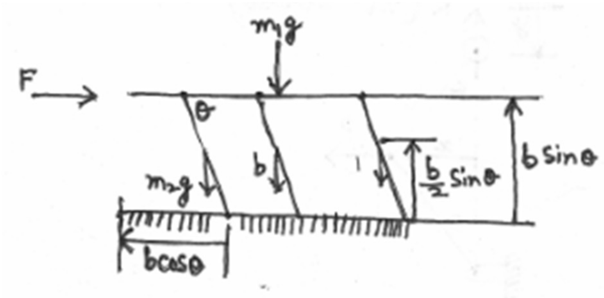
$$I_{max} = 1.7690 \times 10^{-4} + 0.7358 \times 10^{-4}$$

$$\Rightarrow \boxed{I_{max} = 2.5048 \times 10^{-4} \; m^4}$$

$$I_{min} = 1.7690 \times 10^{-4} - 0.7358 \times 10^{-4}$$

$$\Rightarrow \boxed{I_{min} = 1.0332 \times 10^{-4} \; m^4}$$

Q. No. 6 Solution



$m_1 = 80\ kg \quad m_2 = 10\ kg \qquad b = 600\ mm$

(a) Virtual work done by the force F is $F\delta(b\ cos\theta)$

(b) Virtual work done by all the links is $-m_1 g\ \delta(b\sin\theta) - 3m_2\ g\ \delta\left(\frac{b}{2}\sin\theta\right)$

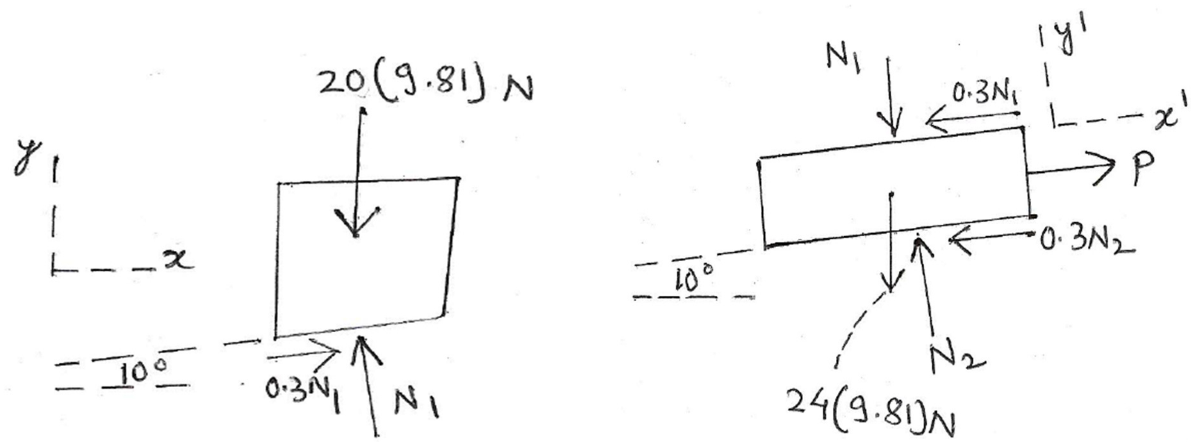
(c) The horizontal force can be obtained by $\delta U = 0$

$$Fb\sin\theta\ \delta\theta = m_1 gb\cos\theta\ \delta\theta + \frac{3}{2}\ m_2\ gb\cos\theta\ \delta\theta$$

$$F = g\cot\theta\ \left(m_1 + \frac{3}{2}\ m_2\right) = 9.81\left(80 + \frac{3}{2}\times 10\right)\cot\theta = 932\cot\theta\ N$$

Q. No. 7 Solution

7 (a)



7 (b)

$20\ kg\ block$

$\sum F_y = 0$:

$N_1 \cos 10^o + 0.3\ N_1 \sin 10^o - 20\ (9.81) = 0$

$N_1(0.985 + 0.0521) = 196.2$

$N_1 = 196.2/1.037 = 189.2\ N$

7 (c)

$24\ kg\ block$

$\sum F_{y'} = 0$: $N_2 - 24(9.81)\cos 10^o - 189.2 = 0$ $N_2 = 421\ N$

$\sum F_{x'} = 0$: $P - 0.3(421) - 0.3(189.2) - 24(9.81)\sin 10^o = 0$

$P = 183.1 + 40.9 = 224\ N$