DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA543: Functional Analysis | Quiz II
Instructor: Rajesh Srivastava | November 8, 2019
Time duration: Two hours | Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

---

1. (a) Let X and Y be two normed linear spaces. If $T : X \to Y$ is a linear map that satisfying $\inf\{\|Tx\| : \|x\| = 1\} > 0$. Does it imply that T is injective? **[1]**

   (b) Let X and Y be two normed linear spaces. Suppose $T \in B(X, Y)$ is open and injective. Does it imply that T is invertible? **[1]**

   (c) Whether the identity linear transformation $I : (l^1, \|\cdot\|_1) \to (l^1, \|\cdot\|_\infty)$ is a closed map? **[1]**

2. Let $T : (L^1[0,1], \|\cdot\|_1) \to (C[0,1], \|\cdot\|_\infty)$ be the linear transformation defined by

$$T(f)(x) = \int_0^x e^{-t^2} f(t)dt.$$

   Show that T satisfies the condition $\frac{1}{3} \leq \|T\| \leq 1$. **[3]**

3. Let X be a Banach space. If the sequence $T_n \in B(X, X)$ converges to T such that T_n^{-1} exists and bounded in $B(X, X)$. Then show that T is invertible. **[2]**

4. Let $T : (C^1[0,1], \|\cdot\|_\infty) \to (C[0,1], \|\cdot\|_\infty)$ be the linear transformation defined by $T(f) = f'$. Denote $M = \{f \in C^1[0,1] : f(0) = 0\}$. Show that $T(M)$ is a closed subspace of $(C[0,1], \|\cdot\|_\infty)$. **[2]**

END