DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA543: Functional Analysis	Quiz II
Instructor: Rajesh Srivastava	November 5, 2020
Time duration: One hour	Maximum Marks: 15

N.B. Answer without proper justification will attract zero mark.

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1. Let c_o be the space all sequences of complex numbers, which are converging to zero. For $(x_n) + c_o \in \ell^\infty/c_o$, define $\|(x_n) + c_o\| = \lim_{n\to\infty} \inf |x_n|$. What is the dimension of the Hamel basis of the quotient space $(\ell^\infty/c_o, \|\cdot\|)$? **[1]**

2. Let X and Y be two normed linear spaces. Suppose $T \in B(X,Y)$ be onto. Define $\tilde{T} : X/\ker T :\to Y$ by $\tilde{T}(x + \ker T) = T(x)$. Does it imply that $\tilde{T}$ is a bounded linear transformation? **[1]**

3. For $(x_n) \in \ell^2$, let $\|(x_n)\| = \sup_{n\in\mathbb{N}} |\sum_{i=1}^{n} \frac{x_i}{i}|$. Show that $\|\cdot\|$ is norm on l^2. Prove/disprove that $(\ell^2, \|\cdot\|)$ is a Banach space. **[4]**

4. Let $X = C[0,1]$. Suppose $g \in X$ has only finitely many zero in $[0,1]$. For $f \in X$, let $\|f\| = \sup |g(t)f(t)|$. Show that $(X, \|\cdot\|)$ is normed linear space but need not be a Banach space. Examine for $(X, \|\cdot\|)$ to be a separable space. **[5]**

5. Let $C_c(\mathbb{R})$ be the class of all compactly supported continuous functions on $\mathbb{R}$. Find all p with $1 \le p \le \infty$ such that T given by $T(f) = \int_{-\infty}^{\infty} f(t)dt$ is continuous linear functional $(C_c(\mathbb{R}), \|\cdot\|_p)$. **[4]**

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