DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA543: Functional Analysis — Quiz I
Instructor: Rajesh Srivastava — September 5, 2019
Time duration: Two hours — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) What is closure of the set $\left\{\left(x, \sin\frac{1}{x}\right) : \ x \neq 0, \ x \in \mathbb{R}\right\}$ in $\mathbb{R}^2$? **1**

   (b) Suppose M is a dense subspace of a normed linear space X. If M is separable, does it imply X separable? **1**

   (c) Whether the set $\{f \in C[0,1] : \|f\|_1 < 1\}$ is bounded in normed linear space $(C[0,1], \|\cdot\|_\infty)$? **1**

2. Let X be a normed linear space. Define $f : X \to \mathbb{R}$ by $f(x) = \frac{1}{1+\|x\|^2}$. Show that f is uniformly continuous on X. **2**

3. For $f \in C^1[0,1]$, define $\|f\| = \|f\|_\infty + \|f'\|_\infty$. Show that $f_n(t) = \frac{e^{-nt^2}}{n} \to 0$ in the space $(C^1[0,1], \|\cdot\|)$. **2**

4. Suppose $\alpha > 0$. For $f \in L^\infty[0,1]$, write $\|f\| = \min\{\|f\|_\infty, \alpha\|f\|_1\}$. Then $\|\cdot\|$ is a norm on $L^\infty[0,1]$ if and only if $\alpha \leq 1$. **3**

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