DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA543: Functional Analysis — Quiz I
Instructor: Rajesh Srivastava — October 16, 2020
Time duration: One hour — Maximum Marks: 15

N.B. Answer without proper justification will attract zero mark.

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1. Let $\alpha > 1$ and $x_n = \dfrac{(\alpha + \frac{1}{n})^{\frac{1}{n}} - \sin n^2}{n}$. Determine all possible p with $1 \leq p \leq \infty$ for which $(x_n) \in l^p$. **[3]**

2. Let $C^2([-1,1])$ be the space of all twice continuously differential function f on $[-1,1]$ such that $f(0) = f'(0) = 0$. Given $f \in \mathbb{C}^2[-1,1]$, define a function $\|\cdot\|$ $C^2([-1,1])$ by $\|f\| = \sum_{i=0}^{2} \|f^{(i)}\|_\infty$. Show that $\|f\| \leq \frac{7}{2}\|f^{(2)}\|_\infty$. **[3]**

3. Let (a_n) be sequence of non-negative real numbers. For each $x = (x_1, x_2, \cdots) \in l^p$ with $1 \leq p < \infty$, define a function $\|\cdot\|$ on l^p by $\|x\| = \left(\sum_{n=1}^{\infty} a_n |x_n|^p\right)^{\frac{1}{p}}$. Determine all possible sequence (a_n) for which $\|\cdot\|$ becomes a norm on l^p. **[3]**

4. Let $X = L^1(\mathbb{R}) \cap L^2(\mathbb{R})$. Given $f \in X$, define a function $\|\cdot\|$ on X by $\|f\| = \min\{2\|f\|_1, \|f\|_2\}$. Prove/disporve that $\|\cdot\|$ is norm on X. **[3]**

5. Let Y be the space of all continuous function f on $\mathbb{R}$ such that for each $\epsilon > 0$ there exists a bounded open set O in $\mathbb{R}$ satisfying $|f(x)| < \epsilon$, whenever $x \in \mathbb{R} \setminus O$. Show that each $f \in Y$ is bounded. Prove/disprove that $(Y, \|\cdot\|_\infty)$ is a Banach space. **[3]**

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