DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA543: Functional Analysis — Mid Semester Exam
Instructor: Rajesh Srivastava — September 17, 2019
Time duration: Two hours — Maximum Marks: 30

N.B. Answer without proper justification will attract zero mark.

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1. (a) If A is closed proper subspace of a normed linear space X, does it imply interior of A is empty? $\boxed{1}$

   (b) Whether the set $\{x \in l^\infty : \|x\|_1 < 1\}$ is separable in l^∞? $\boxed{1}$

   (c) Is it possible that the quotient space l^∞/c_o contains a Schauder basis? $\boxed{1}$

2. Show that $X = \{(x_n) \in l^1 : \sum_{n=1}^{\infty} n|x_n| < \infty\}$ is a proper dense subspace of l^1. $\boxed{4}$

3. Let A be nonempty subset of a normed linear space X and $d(x, A) = \inf\limits_{a \in A} \|x - a\|$. Show that $\bar{A} = A \cup \{x \in X : d(x, A) = 0\}$. $\boxed{3}$

4. Find support of the function $f = \chi_{\left\{\frac{m}{2^n}:\ m\in\mathbb{Z},\text{ and } n\in\mathbb{N}\right\}}$. Whether $\chi_{\text{supp f}} \in \mathcal{R}[0, 1]$? $\boxed{2}$

5. Examine for convergence of the sequence $f_n(t) = \frac{1}{1+nt}$ to 0 in the normed linear space $(C[0, 1], \|\cdot\|_1 + \|\cdot\|_\infty)$. $\boxed{3}$

6. Find all possible $p \geq 1$ such that the sequence $f_n = \chi_{(\sqrt{n}, \sqrt{n+1})} \to 0$ in $L^p(\mathbb{R})$. Does the series $\sum_{n=1}^{\infty} f_n$ converge in $L^p(\mathbb{R})$ for some $p > 1$? $\boxed{4}$

7. Let c be the space of all convergence sequences on $\mathbb{C}$. Prove that the quotient norm on c/c_o is given by $\|\widetilde{(x_n)}\| = \lim\limits_{n\to\infty} |x_n|$. Further deduce that $c/c_o \cong \mathbb{C}$. $\boxed{4}$

8. Show that $\{f \in L^\infty[0, 1] : \int_0^1 f(t)dt = 0\}$ is an infinite dimensional closed subspace of $L^\infty[0, 1]$. $\boxed{3}$

9. Let $1 \leq p < \infty$. For Lebesgue measurable function f on $[0, 1]$, define $\|f\|_p = \left(\int_0^1 |f(t)|^p dt\right)^{\frac{1}{p}}$. Show that $\|\cdot\|_1$ and $\|\cdot\|_2$ are not equivalent. $\boxed{4}$

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