

DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA550: Measure Theory
Instructor: Rajesh Srivastava
Time duration: Two hours

Quiz II
November 4, 2018
Maximum Marks: 15

N.B. Answer without proper justification will attract zero mark.

1. (a) For $n \in \mathbb{N}$, define $f_n = \chi_{(n, n+1)}$. Does there exist a measurable set $E \subset \mathbb{R}$ with $m(E) = \infty$ such that f_n converges to 0 uniformly on E ? **1**
(b) Let $f_n : (\mathbb{R}, M, m) \rightarrow [0, \infty]$ be sequence of measurable functions that converges to f point-wise. If $\int_{\mathbb{R}} f_n dm \leq M < \infty$, does it imply that $\int_{\mathbb{R}} f dm = \lim \int_{\mathbb{R}} f_n dm$? **1**
2. Construct a function $f \in L^1(\mathbb{R}, M, m)$ such that $\lim n^2 m\{x \in \mathbb{R} : |f(x)| \geq n\} = \infty$. **3**
3. Let f be a measurable function on a measure space (X, S, μ) . Suppose there exists an increasing sequence $E_n \in S$ such that $\cup_{n=1}^{\infty} E_n = X$ and $\lim \int_{E_n} |f| d\mu < \infty$. Show that $f \in L^1(X, S, \mu)$. **2**
4. Suppose $f_n, f : (X, S, \mu) \rightarrow [0, \infty]$ are measurable functions such that f_n converges to f point-wise and $f_n \leq f$. Show that $\int_X f d\mu = \lim \int_X f_n d\mu$. **3**
5. Let $f_n : (X, S, \mu) \rightarrow \bar{\mathbb{R}}$ be sequence of measurable functions that f_n increases to f point-wise. If $f, f_n \in L^1(X, S, \mu)$, then show that $\overline{\lim} \int_X f_n d\mu \leq \int_X f d\mu$. **2**
6. Find the $\lim \int_0^{\infty} \frac{n}{n^2 x^2 + e^n \sqrt{x}} dx$. **3**

END