DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA550: Measure Theory	Quiz I
Instructor: Rajesh Srivastava	September 8, 2018
Time duration: Two hours	Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) If A and $A \cup B$ are Lebesgue measurable subsets of $\mathbb{R}$. Is it necessary that B is Lebesgue measurable in $\mathbb{R}$? **[1]**

   (b) Let $O = \cup_{n=1}^{\infty} I_n$, where $\{I_n\}$ is a sequence of pairwise disjoint non-empty open intervals in $\mathbb{R}$ and $m(O) > 1$. Does there exist some $N \in \mathbb{N}$ such that $\sum_{n=1}^{N} l(I_n) > 1$? **[1]**

2. Let $\mathcal{B}_o(\mathbb{R})$ be the $\sigma-$algebra generated by all bounded open intervals in $\mathbb{R}$. Let $\mathcal{B}_1(\mathbb{R})$ be the $\sigma-$algebra generated by all compact sets in $\mathbb{R}$. Show that $\mathcal{B}_o(\mathbb{R}) = \mathcal{B}_1(\mathbb{R})$. **[2]**

3. Let A be a closed subset of $[0, 1]$ that satisfies $A \cap (\alpha, \beta) \neq \emptyset$ for all $\alpha, \beta \in [0, 1]$ with $\alpha < \beta$. Show that $m(A \setminus A^2) = 0$. **[2]**

4. Let A be a subset of $\mathbb{R}$ with $0 < m^*(A) < \infty$. Show that for each $\epsilon > 0$ there exist an open set O containing A and a compact set $K \subset \mathbb{R}$ such that $m(O \setminus K) < \epsilon$. **[2]**

5. Prove that there does not exist a non-zero finite measure μ on the measurable space $(\mathbb{R}, M(\mathbb{R}))$ which is constant on all bounded open interval (a, b) in $\mathbb{R}$ with $a < b$. **[2]**

END