DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA550: Measure Theory — Quiz I
Instructor: Rajesh Srivastava — September 17, 2021
Time duration: One hour — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Let E be a subset of $\mathbb{R}$ such that $m^*(E) = 0$. Does it imply that E is contained in a Borel set B with $m^*(B) = 0$? **1**

   (b) For $A \subseteq \mathbb{R}$, define $\mu_o(A) = \begin{cases} 0 & \text{if } A \text{ is a compact subset of } \mathbb{R}, \\ 1 & \text{if } A \text{ is a non-compact subset of } \mathbb{R}. \end{cases}$
   Whether μ_o is a pre-measure on $\mathcal{B}(\mathbb{R})$ (Borel σ-algebra)? **1**

2. $\overline{\mathcal{A}} = \{E \cap N : E \in \mathcal{A} \text{ and } N \subseteq F \in \mathcal{A} \text{ with } m(F) = 0\}$.

3. Let μ^* be an outer measure on a non-empty set X. Let $F = \{A \subseteq \mathbb{R} : \ \mu^*(A) = 0\}$. Find the σ-algebra generated by F. **2**

4. Let $\{A_n\}_{n=1}^{\infty}$ be a sequence of disjoint open sets in $\mathbb{R}$. Then show that $m^*(\cup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} m^*(A_n)$. **2**

5. Let E be a subset of $\mathbb{R}$. If for every $\epsilon > 0$ there exists a closed interval $\tilde{I}$ of finite length containing E such that $m^*(\tilde{I} \setminus E) < \epsilon$. Then show that E is Lebesgue measurable. **2**

6. Let μ^* be an outer measure generated by a pre-measure μ_o on an algebra $\mathcal{A}$ on $\mathbb{R}$. Let E be μ^* measurable (in Caratheodory's sense). Show that there exists a set $G \in \sigma(\mathcal{A})$ such that $\mu^*(E) = \mu^*(G)$. **2**

END