DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA641: Operator Theory in Hilbert Spaces — Quiz I
Instructor: Rajesh Srivastava — February 21, 2020
Time duration: Two hours — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Let M be subspace of a Hilbert space H and let P_M be a projection map on M. Does it imply that extension of P_M to $\overline{M}$ is an orthogonal projection? **1**

2. Show that a subspace M of Hilbert space H is closed if and only if there exists $f \in H^*$ such that $M = \ker f$. **2**

3. Suppose $T : L^2[0,1] \to L^2[0,1]$ is a linear map defined by $T(f)(x) = \int_0^1 (x-t)f(t)dt$. Show that T is one-one and continuous. **3**

4. Let $\{e_n\}$ be an orthonormal basis for a Hilbert space H. Suppose $T : H \to H$ is linear transformation that satisfying $\sum_{n=1}^{\infty} |\langle Te_n, e_n\rangle| < \infty$. Show that there exists $y \in H$ such that $\langle Tx, y\rangle = \sum_{n=1}^{\infty} \langle Tx, e_n\rangle \overline{\langle Te_n, e_n\rangle}$. Whether such y is unique? **4**

END