Instructions:

1. **Attempt all questions.** There are 25 questions of 2 marks each.

2. **Each question can have one or multiple correct answers.** Write the option(s) (separated by commas) in the space provided next to each question. You must use black or blue pen for writing the option(s).

3. **Evaluation Scheme:** You would be awarded full marks in a question if and only if you write all the correct options and don’t write any of the wrong options. There would not be any negative marking.

4. You would be awarded an F grade either for possessing a mobile phone during the examination, or for adopting any kind of unfair means during the examination.

5. You are allowed to keep the “Cormen et. al: Introduction to Algorithms” book with you. No other books or notes would be allowed to be kept with you. You are also not allowed to interchange the book or any other item with another student during the exam.

6. No discussions or clarification of doubts would be allowed during the exam. Write the answers based on your understanding of the question. In case a question turns out to be wrong, you would be awarded full marks for it irrespective of whether you attempt that question or not.

7. **All the graphs mentioned in this question paper are simple (no self-loops or parallel edges), unless mentioned otherwise.**

Name: 

Roll No: 

Signature of student: 

Signature of invigilator: 

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Q. Consider the label sequences obtained by the preorder, inorder and postorder traversals on a labelled binary tree. Identify the correct statement(s).

(A) The tree can be identified uniquely using only the label sequences of the preorder and postorder traversals.

(B) The tree can be identified uniquely using only the label sequences of the inorder and postorder traversals.

(C) The tree can be identified uniquely using only the label sequences of the inorder and preorder traversals.

(D) The tree can be identified uniquely if and only if the preorder, inorder and postorder traversals of the tree are given.

(E) None of the above.

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Q. The degree sequence of a simple undirected graph is the sequence of the degrees of the vertices in a decreasing order. Which of the following sequences is/are a valid degree sequence of any graph?

(A) 7, 6, 5, 4, 4, 3, 2, 1

(B) 6, 6, 6, 6, 3, 3, 2, 2

(C) 7, 6, 6, 4, 4, 3, 2, 2

(D) 8, 7, 7, 6, 4, 2, 1, 1

(E) None of the above.
Q. Consider an undirected random graph with eight vertices. Given any pair of vertices, the probability that there is an edge between them is $\frac{1}{2}$. What is the expected number of cycles of length three in this graph?

(A) $\frac{1}{8}$
(B) 1
(C) 7
(D) 8
(E) None of the above.

Q. Consider a complete undirected graph with vertex set $\{0, 1, 2, 3, 4\}$. The entry $W_{ij}$ in the matrix $W$ below is the weight of the edge $\{i, j\}$.

$$
W = \begin{pmatrix}
0 & 1 & 8 & 1 & 4 \\
1 & 0 & 12 & 4 & 9 \\
8 & 12 & 0 & 7 & 3 \\
1 & 4 & 7 & 0 & 2 \\
4 & 9 & 3 & 2 & 0
\end{pmatrix}
$$

What is the minimum possible weight of a spanning tree $T$ in this graph such that vertex 0 is a leaf node in the tree $T$?

(A) 7
(B) 8
(C) 9
(D) 10
(E) None of the above.

Q. Consider the following sequence of operations on an unknown data structure, where the number $i$ denotes $\text{insert}(i)$ into the data structure and $*$ denotes a specified operation (as indicated in the options) on the data structure. The data structure is initially empty.

$1, 12, 5, *, 3, 7, *, *, 2, 4, 13, *, 14, 15, *, *, *$

If 1 and 2 are the only remaining elements in the data structure after the above sequence of operations, then the possible choice(s) for the data structure and the $*$ operation is/are:

(A) The data-structure can be a max-heap when the $*$ operation is a delete-max operation.
(B) The data-structure can be a min-heap when the $*$ operation is a delete-min operation.
(C) The data-structure can be a stack when the $*$ operation is a pop operation.
(D) The data-structure can be a queue when the $*$ operation is a dequeue operation.
(E) None of the above.

Q. Consider a scheme for storing binary trees in an array $X$ as follows: Indexing of $X$ starts at 1 instead of 0. The root is stored at $X[1]$. For a node stored at $X[i]$, the left child, if any, is stored in $X[2i]$ and the right child, if any, in $X[2i + 1]$. To be able to store any binary tree with $n$ vertices, the size of $X$ has to be at least:

(A) $\log n$
(B) $n - 1$
(C) $n + 1$
(D) $2n + 1$
(E) None of the above.
Q. An algorithm takes as input a balanced binary search tree with n leaf nodes and computes the value of a function \( g(x) \) for each node \( x \) in the tree. If the cost of computing \( g(x) \) is 
\[
\min\{\text{number of leaf nodes in left sub-tree of } x, \text{ number of leaf nodes in right sub-tree of } x\},
\]
then the worst-case time complexity of the algorithm is:

(A) \( \Theta(n) \)
(B) \( \Theta(n \log n) \)
(C) \( \Theta(n^2) \)
(D) \( \Theta(2^n) \)
(E) None of the above.

Q. A complete \( n \)-ary tree is a tree in which each node has either \( n \) children or no children. Let \( I \) be the number of internal nodes (any node except the leaves is an internal node) and \( L \) be the number of leaves in a complete \( n \)-ary tree. If \( L = 41 \) and \( I = 10 \), what is the value of \( n \)?

(A) 3
(B) 4
(C) 5
(D) 6
(E) None of the above.

Q. Let \( G = (V, E) \) be a connected undirected graph on \( n \) vertices and \( 2n - 2 \) edges, such that the edges of \( G \) can be partitioned into two edge-disjoint spanning trees. Which of the following is/are true for \( G \)?

(A) For every subset \( U \subseteq V \) with \( k \) vertices, the sub-graph induced by \( U \) has at most \( 2k - 2 \) edges.
(B) The minimum cut in \( G \) has at least two edges.
(C) There are at least two edge-disjoint paths between every pair of vertices.
(D) There are at least \( n - 1 \) cycles in \( G \).
(E) None of the above

Q. The best algorithm (with respect to the worst-case time complexity) for finding the number of connected components in an undirected graph on \( n \) vertices and \( m \) edges has the following worst-case time complexity:

(A) \( \Theta(m) \).
(B) \( \Theta(n) \).
(C) \( \Theta(m + n) \).
(D) \( \Theta(mn) \)
(E) None of the above.
Q. We are given a directed graph in which every vertex has exactly seven edges coming into it. What can one always say about the number of edges coming out of its vertices?

(A) Exactly seven edges come out of every vertex.
(B) There exists some vertices such that exactly seven edges come out from each of them.
(C) There exists a vertex having at least seven edges coming out of it.
(D) The number of edges coming out of every vertex is odd.
(E) None of the above.

Q. For an even integer $n$, consider an undirected graph $G$ with $n$ vertices. If the degree of each vertex of $G$ is at least $\frac{n}{2}$, then identify the statement(s) which is/are true.

(A) $G$ must be connected.
(B) $G$ must be a tree.
(C) $G$ must contain a cycle.
(D) $G$ must have a perfect matching.
(E) None of the above.

Q. Suppose an AVL tree is constructed by inserting 15, 20, 26, 7, 18, 27 and 28 in this order. Which of the following statement(s) is/are correct about this AVL tree?

(A) The root node contains 20 and there are a total of four leaves in the tree.
(B) The root node contains 26 and there are a total of four leaves in the tree.
(C) The children of the node containing 20 are the nodes containing 15 and 26.
(D) The children of the node containing 27 are the nodes containing 26 and 28.
(E) None of the above

Q. An undirected graph $G = (V, E)$ contains $n > 2$ vertices named $v_1, v_2, \ldots, v_n$. Two vertices $v_i, v_j$ are connected if and only if $0 < |i - j| \leq 2$. Each edge $(v_i, v_j)$ is assigned a weight $i + j$. What is the cost of the minimum spanning tree (MST) of such a graph with $n$ vertices?

(A) $n^2 - n - 1$
(B) $n^2 - n$
(C) $n^2 - n + 1$
(D) $3n + 1$
(E) None of the above
Q. Let $T$ be a depth-first tree of an undirected connected graph $G$. Assume that the vertices $u$ and $v$ are the leaves of $T$, and that the degrees of both $u$ and $v$ in $G$ are at least 2. Identify the correct statement(s).

(A) There must exist a vertex adjacent to both $u$ and $v$ in $G$.
(B) There must exist a vertex $w$ whose removal disconnects $u$ and $v$ in $G$.
(C) There must exist a cycle in $G$ containing both $u$ and $v$.
(D) There must exist a cycle in $G$ containing $u$ and all its neighbours in $G$.
(E) None of the above.

Q. Identify the relationship(s) that hold(s) between the number of cycles $C$ and the number of back edges $B$ found during the DFS of an undirected graph.

(A) $C \geq B$
(B) $C = B$
(C) $C > B$
(D) $C < B$
(E) None of the above.

Q. If we want to implement Dijkstra’s shortest path algorithm on an unweighted graph $G = (V,E)$ so that it runs in $O(|V| + |E|)$-time, identify the data structure(s) that can be used to replace the Fibonacci heap that is used in the weighted case.

(A) A queue
(B) A stack
(C) A B-tree
(D) An AVL-tree
(E) None of the above.

Q. Identify the data-structure(s) that can be used to implement each of insert, delete and search operations in $O(\log n)$ time, when the data structure contains $n$ keys.

(A) B-tree
(B) AVL tree
(C) Red-Black tree
(D) $B^+$-tree
(E) None of the above
Q. Consider an undirected graph \( G = (V, E) \) with \( V = \{1, 2, 3, 4, 5, 6\} \) and \( E = \{\{1, 2\}, \{1, 4\}, \{1, 6\}, \{2, 3\}, \{2, 4\}, \{3, 5\}\} \). Which of the following is/are a valid sequence of vertices *discovered* by a DFS over this graph, if the search starts at vertex 1?

(A) 1, 6, 5, 3, 4, 2
(B) 1, 2, 3, 5, 6, 4
(C) 1, 4, 6, 5, 3, 2
(D) 1, 2, 3, 4, 5, 6
(E) None of the above.

Q. Consider an undirected graph \( G = (V, E) \) with \( V = \{1, 2, 3, 4, 5, 6\} \) and \( E = \{\{1, 2\}, \{1, 4\}, \{1, 6\}, \{2, 3\}, \{2, 4\}, \{3, 5\}\} \). Which of the following is/are a valid sequence of vertices *discovered* by a BFS over this graph, if the search starts at vertex 1?

(A) 1, 6, 5, 3, 4, 2
(B) 1, 2, 3, 5, 6, 4
(C) 1, 4, 6, 5, 3, 2
(D) 1, 2, 3, 4, 5, 6
(E) None of the above.

Q. Consider a directed graph \( G = (V, E) \) with \( V = \{1, 2, 3, 4, 5, 6\} \) and \( E = \{\{1, 2\}, \{1, 4\}, \{1, 6\}, \{2, 3\}, \{2, 4\}, \{3, 5\}\} \), where \( \{i, j\} \) indicates the existence of an edge from \( i \) to \( j \). Which of the following is/are a valid topological order of the vertices in \( V \)?

(A) 1, 2, 3, 4, 5, 6
(B) 1, 3, 2, 4, 5, 6
(C) 1, 3, 2, 4, 6, 5
(D) 1, 2, 3, 4, 6, 5
(E) None of the above


[Note: Follow the definition of *minimum degree of a B-tree* as given in Cormen et al. Moreover, the *insert* and *delete* operations should be implemented as described in Cormen et al.]

(A) 4
(B) 8
(C) 10
(D) 11
(E) None of the above.
Q. Suppose we execute any maximum flow algorithm on the following directed weighted graph $G = (V, E)$ with vertex $P$ as the source and $U$ as the destination. What is the value of the maximum flow in this graph?

**Note:** Assume that the element $(X, Y, i)$ in $E$ indicates the existence of a directed edge from $X$ to $Y$ with a capacity equal to $i$.

$V = \{P, Q, R, S, T, U\}$.

$E = \{(P, Q, 1), (P, S, 6), (P, T, 7), (Q, R, 1), (Q, S, 4), (T, U, 2), (R, U, 1), (S, U, 2)\}$.

(A) 1
(B) 2
(C) 3
(D) 4
(E) None of the above.

Q. Suppose we execute Dijsktra’s shortest path algorithm (as given in Cormen et al.) on the following directed weighted graph $G = (V, E)$ with vertex $P$ as the source. Identify the possible order(s) of vertices for which the shortest path distance from $P$ gets finalized, i.e., it doesn’t change afterwards.

**Note:** Assume that the element $(X, Y, i)$ in $E$ indicates the existence of a directed edge from $X$ to $Y$ with an edge-weight equal to $i$.

$V = \{P, Q, R, S, T, U\}$.

$E = \{(P, Q, 1), (P, S, 6), (P, T, 7), (Q, R, 1), (Q, S, 4), (T, U, 2), (R, U, 1), (S, U, 2)\}$.

(A) P, Q, R, S, T, U
(B) P, Q, R, U, S, T
(C) P, Q, R, U, T, S
(D) P, Q, T, R, U, S
(E) None of the above.

Q. Suppose we execute Bellman-Ford algorithm (as given in Cormen et al.) on the following directed weighted graph $G = (V, E)$ with vertex $P$ as the source. Identify the possible order(s) of vertices for which the shortest path distance from $P$ gets finalized, i.e., it doesn’t change afterwards.

**Note:** Assume that the element $(X, Y, i)$ in $E$ indicates the existence of a directed edge from $X$ to $Y$ with an edge-weight equal to $i$.

$V = \{P, Q, R, S, T, U\}$.

$E = \{(P, Q, 1), (P, S, 6), (P, T, 7), (Q, R, 1), (Q, S, 4), (T, U, 2), (R, U, 1), (S, U, 2)\}$.

(A) P, Q, R, S, T, U
(B) P, Q, R, U, S, T
(C) P, Q, R, U, T, S
(D) P, Q, T, R, U, S
(E) None of the above.