CHAPTER 14

Experimental Estimation of Dynamic Parameters of Bearings, Dampers and Seals

One of the important factors governing the vibration characteristics of rotating machinery is the effective dynamic stiffness of the supports and similar components as seen by the rotor. The dynamic stiffness of the support is determined by the combined effects of flexibility of the bearing, the bearing pedestal assembly (bearing housing) and the foundations on which the pedestal is mounted. For the case of turbo generator rotors mounted on oil-film bearings might be three times more flexible as compared to pedestals and foundations. In Chapter 3, various kinds of bearings, dampers, and seals were described and the main focus of the chapter was to outline theoretical methods of calculation of dynamic parameters for bearings, dampers, and seals. Practical limitations of such procedures would be outlined subsequently. In the present chapter, the focus is the same of obtaining the dynamic parameters of bearings, dampers, and seals; however, methods involve the experimental estimation based on the force-response information of rotor systems. These methods are broadly classified based on the force given to the system, i.e., the static and dynamic forces. The static method can be used for obtaining stiffness coefficients only; however, dynamic methods can be used to obtain both the stiffness and damping coefficients. Initially, these methods are described for the rigid rotors mounted on flexible bearings. Subsequently, literatures survey is presented for the methods extended for more real rotors by considering shaft and bearings both as flexible and more general modelling technique, i.e., finite element methods. Various methods are compared for their merits and demerits in terms of the simplicity of applications, accuracy, consistency, robustness, versatility, etc. Experimental considerations for designing the test rig, types of vibration measurements, suitable condition at which measurement of vibration signals to be taken, processing of raw vibration signals, etc. are also addressed.

Various Machine Elements having similar dynamic characteristics: In many industries (such as machine tools, automobile and aero industries and power plants), ever-increasing demand for high power and high speed with uninterrupted and reliable operation the accurate prediction and control of the dynamic behaviour (unbalance response, critical speeds and instability) increasingly important. The most crucial part of such large turbo-machinery is the machine elements that allow relative motion between the rotating and the stationary machine elements i.e. the bearings. In the present chapter the term bearings refer to rolling element bearings, fluid-film bearings (journal and thrust; hydrostatic (external pressurisation), hydrodynamic, hybrid (combination of the hydrostatic and
hydodynamic), gas-lubricated and squeeze-film), magnetic, foil bearings together with dampers and seals, since their dynamic characteristics have some common features.

**Developments in Bearing Technology:** A hybrid bearing is a fluid-film journal bearing which combines the physical mechanisms of both hydrostatic and hydrodynamic bearings. Hybrid bearings are being considered as alternatives to rolling-element bearings for future cryogenic turbo-pumps. The hydrostatic characteristics of a hybrid bearing allow it to be used with low viscosity fluids that could not adequately carry a load with purely hydrodynamic action. A compliant surface foil bearing consist a smooth top foil that provides the bearing surface and a flexible, corrugated foil strip formed by a series of bumps that provides a resilient support to this surface. Compared to conventional journal bearings, the advantage offered by the compliant surface foil bearing include its adaptation to shaft misalignment, variations due to tolerance build-ups, centrifugal shaft growth and differential expansion. Apart from the use of the conventional rolling element and fluid-film bearings and seals, which affects the dynamics of the rotor, squeeze film and magnetic bearings are often used to control the dynamics of such systems. Squeeze-film bearings are, in effect, fluid-film bearings in which both the journal and bearing are non-rotating. The ability to provide damping is retained but there is no capacity to provide stiffness as the latter is related to journal rotation. They are used extensively in applications where it is necessary to eliminate instabilities and to limit rotor vibration and its effect on the supporting structures of rotor-bearing systems, especially in aeroengines. In recent years, advanced development of electromagnetic bearing technology has enabled the active control of rotor bearing systems. In particular the electromagnetic suspension of a rotating shaft without mechanical contact has allowed the development of supercritical shafts in conjunction with modern digital control strategies. With the development of *smart fluids* (for example electro and magneto-rheological fluids) now new controllable bearings are in the primitive development stage.

**Limitations of Theoretical Predictions:** Historically, the theoretical estimation of the dynamic bearing characteristics has always been a source of error in the prediction of the dynamic behaviour of rotor-bearing systems. Obtaining reliable estimates of the bearing operating conditions (such as static load, temperature, viscosity of lubricant, geometry and surface roughness) in actual test conditions is difficult and this leads to inaccuracies in the well-established theoretical bearing models. Consequently, physically meaningful and accurate parameter identification is required in actual test conditions to reduce the discrepancy between the measurements and the predictions. The experimental methods for the dynamic characterisation of rolling element bearings, fluid-film bearings, dampers, magnetic bearings and seals have some similarities. In general the methods require input signals (forces) and output signals (displacements/velocities/accelerations) of the dynamic system to be measured, and the unknown parameters of the system models are calculated by means of input-output
relationships. The theoretical procedures and experimental measurements depend upon whether the bearing is investigated in isolation or installed in a rotor-bearing system.

14.1 Previous Literature Reviews and Surveys

The influence of bearings on the performance of rotor-bearing systems has been recognised for many years. One of the earliest attempts to model a journal bearing was reported by Stodola (1925) and Hummel (1926), they represented the fluid-film as a simple spring support, but their model was incapable of accounting for the observed finite amplitude of oscillation of a shaft operating at critical speed. Concurrently, Newkirk (1924 and 1925) described the phenomenon of bearing-induced instability, which he called oil whip, and it soon occurred to several investigators that the problem of rotor stability could be related to the properties of the bearing dynamic coefficients. Ramsden (1967-68) was the first to review the papers on the experimentally obtained journal bearing dynamic characteristics. From the designer’s perspective he concluded that a designer would require a known stiffness and good vibration damping of bearings. Since most of the data available at that time was from experiments only, so he stressed for accurate scaling laws to be evolved to avoid the need to do expensive full-scale tests. In mid seventies Dowson and Taylor (1980) conducted a survey of the state of knowledge in the field of bearing influence on rotor dynamics. They appreciated that a considerable amount of literature was available on both rotor dynamics and fluid-film bearings but relatively few attempts had been made to integrate the individual studies of rotor behaviour in the field of dynamics and of dynamic characteristics of bearings in the field of tribology. Several conclusions and recommendations were made by them, most importantly: (a) the need for experimental work in the field of rotor dynamics to study the influence of bearings and supports upon the rotor response, in particular for full scale rotor systems, (b) additional theoretical studies to consider the influence of thermal and elastic distortion, grooving arrangements, misalignment, cavitation and film reformation.

A large amount of literature is available on the theoretical calculation of the dynamic characteristics of variety of bearings (rolling element (Palmgren, 1959; Ragulskis, 1974; Gargiulo, 1980; Changsen, 1991; Gupta 1984; Harris, 2000) fluid-film (Pinkus and Sternlicht, 1961; Smith, 1969; Hamrock, 1994), magnetic bearings (Schweitzer et al., 1994) and seals (Black, 1969; Childs, 1993)). Among the various bearings available fluid-film bearings, especially hydrodynamic cylindrical-journal bearings, attracted the most interest of practicing engineers and researchers. This is because of their relatively simplicity in the geometry and consequently in its analysis and experimental rig fabrication along with its general representation in mathematical modelling among the all other classes of the bearings. Lund (1980, 1987) reviewed the concept of dynamic coefficients for fluid-film journal bearings.
Lund (1979) gave a review on the theoretical and experimental methods for the determination of the fluid-film bearing dynamic coefficients. For experimental determination of the coefficients, he suggested the necessity of accounting for the impedance of the rotor and categorised the experimental methods based on the excitation method used (for example: static load and dynamic force i.e. harmonic, transient and random). He observed the general validity of the theory by experimental results, but it was not unusual to find discrepancies of 50 percent or more in a point-by-point comparison. Moreover, the experimental static locus did not coincide with the theoretical one and it was difficult to analyse the degree to which the result might be affected by measurement tolerances and by uncertainties in establishing test parameters. He concluded that there was little point in refining the dynamic analysis until better agreement between theory and measurements had been obtained on the performance of static bearings. Stanway et al. (1979a) gave a brief appraisal of bearing dynamic testing methods. They stressed the need for development of experimental method that must be capable of use with the rotor-bearing system being run under normal operating conditions and place some confidence bounds on the estimates. Stone (1982) gave the state of the art in the measurement of the stiffness and damping of rolling element bearings. The work of three groups (Leuven, Aachen and Bristol) was summarised and essentially concentrated on taper roller bearings and angular contact bearings. Special rigs were built for excitation and response measurements of the rotor. With simple physical models, the bearing coefficients were extracted. He concluded that the most important parameters influence the bearing coefficients were type of bearing, axial preload, clearance/interference, speed, lubricant and tilt (clamping) of the rotor. Kraus et al. (1987) compared different methods (both theoretical and experimental) to obtain axial and radial stiffness of rolling element bearings and showed a considerable amount of variation by using different measurement methods. Someya (1989) compiled extensively both analytical as well as experimental results (static and dynamic parameters) for various fluid-film bearing geometries (for example 2-axial groove, 2-lobe, 4 & 5-pad tilting pad). Rouch (1990) reviewed briefly the theoretical and experimental developments in squeeze-film bearing dynamic coefficients estimation.

Goodwin (1989 and 1991) reviewed the experimental approaches to rotor support impedance measurement (the term impedance used by Goodwin and several others is confusing since the definition of mechanical impedance is the ratio of force excitation to velocity response (Maia et al. 1997), more appropriately it should be stiffness and damping coefficients). He particularly emphasised the measurement of fluid-film bearing dynamic coefficients and categorised the identification methods by the way in which the loading was applied (static load and dynamic force i.e. using vibrator, unbalance or transient force). He concluded that measurements made by multi-frequency test signals provide more reliable data, although all measurement methods yield coefficients values which agree with theoretical predictions to within about 20 percent in general, and
all methods have a significant scatter of results associated with their use. Childs (1993) gave a comprehensive survey of the rotordynamic experimental data for liquid and gas annular seals and turbines and pump impellers along with its geometry and operating conditions. In 1997 Swanson and Kirk presented a survey in tabular form of the experimental data available in the open literature for fixed geometry hydrodynamic journal bearings. They categorised the literature based on bearing type, bore diameter, length to diameter ratio and type of data available (i.e. static parameters such as film pressure, bearing temperature and shaft position; dynamic parameters such as stiffness, damping and added-mass coefficients). Tiwari et al. (2004, 2005) gave a review of experimental identification of rotordynamic parameters of bearings and seals, respectively. Major emphasis was given to the vibration based identification methods. The review encompassed descriptions of experimental measurement techniques, mathematical modelling, parameter extraction algorithms and uncertainty in the estimates applied to a variety of bearings/seals. The experimental techniques included descriptions of test rigs, instrumentation for data collection and methods and types of data collection from the test rigs. The parameter extraction algorithms included the descriptions of governing equations of the rotor-bearing system and identification methods both in time and frequency domains. The uncertainty in the bearing dynamic parameters included both due to numerical calculations and due to the measurements. The identification methods were classified based on excitation methods used to excite the system. The review included a variety of bearings, seals and similar components, which play an active role between the rotating and stationary parts. Based on the state of the art in the bearings/seals identification, conclusions were made and future directions were suggested. A look-up table of a summary of papers on the experimental bearing/seal dynamic identification was provided.

The theoretical models available for predicting the rotor support stiffness are insufficiently accurate. It is for this reason that designers of high-speed rotating must rely on empirically derived values (i.e. experimental) for support stiffness and damping in their design calculations. The present literature on the experimental identification of bearing dynamic parameters can be classified based on the following categories:

(a) **Methods using different excitation devices**: static load, vibrator, unbalance mass (synchronous, anti-synchronous or asynchronous), impact hammer (or sudden release of load), or system’s inherent unknown forces (i.e. residual unbalance, misalignment, rubbing between rotor and stator, aerodynamic forces, oil-whirl, oil-whip or due to instability).

(b) **Types of forcing**: Incremental static, sinusoidal, compound sinusoidal, periodic (multi-sine), step-function, impulse (rap), random, pseudo-random binary sequence (PRBS) or Schroeder-phased harmonic signal (SPHS).

(c) **Location of excitation**: On the journal/rotor or on the floating bearing bush (housing).
(d) **Frequency of excitation**: Synchronous or asynchronous (both in magnitude and direction) with the frequency of rotation of the rotor.

(e) **Use of identified parameters**: Response prediction at design/improvement stage (off-line methods) or for controlling vibration and condition monitoring (on-line methods).

(f) **Bearing model**: Linear without or with frequency (external excitation frequency &/or rotational frequency of the rotor) dependent (four, eight or twelve coefficients) or non-linear (amplitude dependent).

(g) **Type of perturbation**: Controlled (calibrated) displacement or force perturbation.

(h) **Parameter estimation domain**: Time or Frequency domain.

(i) **Type of bearing**: Rolling element bearings, fluid-film bearings, foil bearings, magnetic bearings or seals.

(j) **Rotor model**: Rigid or Flexible.

(k) **Number of bearings**: One, two identical, two or more than two different bearings.

(l) **Co-ordinate system used**: Real or complex (stationary or rotating).

(m) **Size of the bearing**: Small or large-scale bearings working in controlled laboratory environment or bearings working in actual industrial environment (for example: bearings of a turbo-generator).

Since the above classification has some overlap among the different categories, the present chapter gives a descriptions of literatures based on mainly one type of category (i.e. the methods using different excitation devices) with the acknowledgements to the other categories whenever it is appropriate.

### 14.2 Basic Concepts and Assumptions of Bearing Models

For a given bearing and rotational speed, from lubrication theory the reaction forces on the journal from the lubricant film are functions of the displacements of the journal from bearing center and of the instantaneous journal center velocities and accelerations. Hence, for small amplitude motions, measured from the static equilibrium position (see Figure 14.1) of the journal \((u_0, v_0)\), a first order Taylor series expansion yields

\[
\begin{align*}
\mathcal{R}_x &= R_{x0} + k_{xx}x + k_{xy}y + c_{x\dot{x}}\dot{x} + c_{y\dot{y}}\dot{y} + m_{x\ddot{x}}\ddot{x} + m_{y\ddot{y}}\ddot{y} \\
\mathcal{R}_y &= R_{y0} + k_{yx}x + k_{yy}y + c_{x\dot{x}}\dot{x} + c_{y\dot{y}}\dot{y} + m_{x\ddot{x}}\ddot{x} + m_{y\ddot{y}}\ddot{y}
\end{align*}
\]

(14.1)

with
and analogously the remaining bearing dynamic coefficients can be defined. In matrix form of equation (14.1) all diagonal terms are called direct coefficients and off-diagonal terms are called cross-coupled. The latter terms arise due to the fluid rotation within the bearing. $R$ is the reaction force of fluid film on the journal, $f$ is external excitation force on the journal, $m$ is the journal mass, $u_0$ and $v_0$ are the static equilibrium position of the journal from bearing center, $x$ and $y$ are the displacements of the journal from its static equilibrium position, $\dot{x}$ and $\dot{y}$ are the instantaneous journal center velocities and $\ddot{x}$ and $\ddot{y}$ are the instantaneous journal center accelerations, in the vertical and horizontal directions respectively. The “dot” indicates the time derivatives and $k_{ij}$, $c_{ij}$ and $m_{ij}$ ($i, j = x, y$) are bearing stiffness, damping and added-mass (also termed as virtual fluid-film mass or inertia) coefficients respectively. The indices of the stiffness, damping and added-mass coefficients have the following significance: the first index gives the direction of loading which produces elastic (damping/inertia) force and the second index gives the direction of the displacement (velocity/acceleration). Because $(u_0, v_0)$ is the equilibrium position, then $R_x = 0$ while $R_y$ equals to the static load, $W$.

The equilibrium position depends on a unique value of the dimensionless Sommerfeld number ($S = (\mu \omega RL/W)(R/c_r)^2(L/D)^2$), where $\mu$ is the lubricant viscosity, $\omega$ is the journal rotational speed, $D$ is the bearing bore, $R$ is the journal radius, $L$ is the bearing length and $c_r$ is the bearing radial clearance. The Sommerfeld number, $S$, defines the operating conditions (speed, lubricant viscosity,
static load and geometry). The dynamic coefficients are evaluated for a particular static equilibrium position, which is a function of the Sommerfeld number, $S$. This means that for a given application, they are functions of rotor speed. Moreover, bearing dynamic coefficients could be external excitation frequency, $\Omega$, dependent.

It should be noted that the equation (14.1) is a complete form of linearised fluid-film dynamic equation and it contains twelve stiffness, damping and added-mass coefficients. Consistent with the assumptions inherent in reducing the Navier-Stokes equations to the Reynolds equation, the conventional laminar, thin film lubrication theory ignores the inertia forces in the fluid-film (Pinkus and Sternlicht, 1961; Schlichting, 1960). This is theoretically justified for small values of the Reynolds number (of the order of 1). On the other hand, the assumption of laminar flow ceases to be valid when there is transition to either Taylor vortex flow or to turbulence flow which, for fluid-film cylindrical journal bearing, occurs at a Reynolds number of approximately 1000 to 1500. Thus, there is an intermediate range, say for values of Reynolds number of the order of 100, where added-mass effects may become noticeable (several times the mass of the journal itself) without affecting the assumption of laminar flow. The added-mass coefficients represent the mass of the bearing fluid-film (Reinhardt and Lund, 1975) but are significant only in exceptional cases and in most analyses the added-mass of the bearing film are ignored. The stiffness and damping coefficients can be obtained by a finite difference solution of the perturbed Reynolds equation (Lund and Thomsen, 1978).

From lubrication theory (without inertia effect) damping coefficients are symmetric but stiffness coefficients are not. Therefore principal directions do not exist (as against it was assumed by Hagg and Sankey (1956) and Duffin and Johnson (1966-67)), and in the experimental determination of the coefficients, it is necessary to obtain two independent sets of amplitude-force measurements. Lund (1987) emphasised the experimental measurement of the bearing coefficients and established more uniform agreement with analytical calculations by considering the influence of thermal and elastic deformations and practical problems of manufacturing and operating tolerances of bearing geometry, clearance and lubricant viscosity. Although the load-displacement characteristics of a journal bearing is evidently non-linear, the concept of linear dynamic coefficients is still used for modern rotor dynamic calculations for unbalance response, damped natural frequencies and stability since experience has demonstrated the usefulness of the coefficients.

Hydrostatic, hybrid (San Andres, 1990) and gas bearings fluid-film reaction forces, for eccentric journal position are modelled in a similar way to equation (14.1). Since rolling element bearings can allow both radial and axial reaction forces, together with a reaction moment in the radial direction, they have radial and axial-force and radial-moment dynamic coefficients (Lim and Singh, 1990).
Because of the difficulty in the accurate measurement of angular displacements the rolling element bearings linear radial dynamic coefficients are modelled in a similar way to equation (14.1) with negligible added-mass coefficients.

For squeeze-film bearings the governing equation for fluid-film reaction force is of a similar form to equation (14.1) with negligible stiffness coefficients and no static load.

\[
\mathbf{R}_x = m_{xx} \ddot{x} + m_{xy} \ddot{y} + c_{xx} \dot{x} + c_{xy} \dot{y}
\]

\[
\mathbf{R}_y = m_{yx} \ddot{x} + m_{yy} \ddot{y} + c_{yx} \dot{x} + c_{yy} \dot{y}
\]  

(14.3)

Bulk-flow versions of Navier-Stokes equations are normally used for seal analysis (Black, 1969; Childs, 1993). The dynamic equation of a seal’s fluid-film reaction forces has a similar form as equation (14.1) and is normally modelled as

\[
\mathbf{R}_x = k_d x + k_c y + c_d \dot{x} + c_c \dot{y} + m_d \ddot{x}
\]

\[
\mathbf{R}_y = -k_c x + k_d y - c_c \dot{x} + c_d \dot{y} + m_d \ddot{y}
\]  

(14.4)

where subscripts \(d\) and \(c\) represent the direct and cross-coupled terms. The cross-coupled terms arise due to fluid rotation within the seal. The coefficient \(m_d\) accounts for the seal’s added-mass. This model is valid for small motion about a centered position and stiffness and damping matrices have skew-symmetric properties. Theoretically dynamic coefficients are obtained from the first-order perturbation of the bulk-flow governing equations. This model is valid for hybrid bearing and with added-mass matrix as skew-symmetric form for the turbines and pump impellers.

Mittwollen et al. (1991) showed theoretically and experimentally that hydrodynamic thrust bearings, which are often treated as an axial support, might effect the lateral vibration of a rotor-bearing system. Suppose no axial force is present, then the resulting reaction moments of a thrust bearing can be written as (Jiang and Yu, 1999 and 2000)

\[
\mathbf{R}_x = k_{d\theta} \theta + k_{d\psi} \psi + c_{d\theta} \dot{\theta} + c_{d\psi} \dot{\psi}
\]

\[
\mathbf{R}_y = k_{c\theta} \theta + k_{c\psi} \psi + c_{c\theta} \dot{\theta} + c_{c\psi} \dot{\psi}
\]  

(14.5)
where \( k_{x\theta} \) etc. represent moment dynamic coefficients of the thrust bearing, and \( \theta \) and \( \psi \) are angular displacements (slopes) in \( x \) and \( y \) directions, respectively.

For an active magnetic bearing the magnetic force can be written in the linearised form as (Lee et al., 1996)

\[
\begin{align*}
\mathfrak{R}_x &= k_{ix} i_x(t) + k_{x\theta} \theta(t) \\
\mathfrak{R}_y &= k_{iy} i_y(t) + k_{y\theta} \psi(t)
\end{align*}
\]

(14.6)

where \( k_i \) and \( k_i \) are the current and position stiffness coefficients respectively, \( i(t) \) is the control current and \( x(t) \) and \( y(t) \) are the rotor displacements in the vertical and horizontal directions, respectively.

All of the bearing models discussed thus far are linearised models. Few researchers have considered non-linear bearing models and these will be described in appropriate places. The present literature survey is aimed at the review of experimental methods for determination of the rotordynamic parameters of the bearings and related similar components in rotor-bearing systems. It is hoped, it will be useful to both to practising engineers for simple experimental determination of these parameters with associated uncertainty and to researchers in this field to have an idea of the diverse methods available and their limitations so as to develop improved methods.

### 14.3 Abstract definition of the Identification

In actual test conditions obtaining reliable estimates of the bearing operating conditions is difficult and this leads to inaccuracies in the well-established theoretical bearing models. To reduce the discrepancy between the measurements and the predictions physically meaningful and accurate parameter identification is required in actual test conditions. Inverse engineering problems in structural dynamics involves the identification of system model parameters by knowing the response and force information. This is called modal testing (Ewins, 1984) as shown in Figure 14.2. The present inverse engineering problem of identifying bearing dynamic parameters, with partial knowledge of the system parameters (i.e., of the beam model) along with the force and corresponding the response (displacements/velocities/accelerations), falls under the grey system, which is called the model updating (Friswell and Mottershead, 1995).
It is possible to determine all four stiffness coefficients (i.e., $k_{xx}, k_{yy}, k_{xy}$, and $k_{yx}$) of the bearing oil film by application of static loads only. Unfortunately, this method of loading does not enable the oil-film damping coefficients to be determined. The exact operating position of the shaft center on a particular bearing depends upon the Sommerfeld number. Because the bearing oil-film coefficient are specific to a particular location of shaft center on the static locus as shown in Figure 14.1. A static load must first be applied in order to establish operation at the required point on the locus. The next step is to apply incremental loads in both the horizontal and vertical directions, which will cause changes in the journal horizontal and vertical displacements relative to the bearing bush (or more precisely with respect to its static equilibrium position). By relating the measured changes in displacements to the changes in the static load, it is possible to determine four-stiffness coefficients on the bearing oil film. We have increments in fluid-film forces as

$$f_x = k_{xx}x + k_{xy}y; \quad \text{and} \quad f_y = k_{yx}x + k_{yy}y$$

(14.7)

where $x$ and $y$ are the journal displacement in horizontal and vertical directions, respectively (with respect to the static equilibrium position for a particular speed). If the displacement in the vertical direction ($y$-direction) is made to zero by application of suitable loads $f_x$ and $f_y$, then

$$k_{xx} = f_x / x; \quad k_{yx} = f_y / x$$

(14.8)

Similarly, if the displacement in the horizontal direction ($x$-direction) is made zero then

$$k_{xy} = f_x / y; \quad k_{yy} = f_y / y$$

(14.9)
Determination of the oil-film coefficient in this way necessitates a test rig, which is capable of applying loads to the journal in both the horizontal and vertical directions. The method is somewhat tedious in the experimental stage since evaluation of the required loads to ensure zero change in displacement in one or other direction is dependent on the application of trial loads.

**Alternative method** (i) – Instead of applying loads in both $x$ and $y$ directions, to ensue zero displacements in one of these directions, it is easier to simply apply a load in one direction only and measure resulting displacements in both directions. Equations (14.7) can be written as

\[
\{f\} = [K]\{d\} \tag{14.10}
\]

with

\[
\{f\} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}; \quad [K] = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}; \quad \{d\} = \begin{bmatrix} x \\ y \end{bmatrix}
\]

If $[K]$ matrix is inverted then equation (14.4) can be written as

\[
\{d\} = [\alpha]\{f\} \tag{14.11}
\]

with

\[
[K]^{-1} = [\alpha] = \begin{bmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{bmatrix}
\]

where quantities $\alpha_{xx}$, $\alpha_{xy}$ etc. are called the fluid-film influence coefficients. If the force in the $y$-direction is zero then

\[
\alpha_{xx} = \frac{x}{f_x}; \quad \alpha_{yy} = \frac{y}{f_y} \tag{14.12}
\]

Similarly, when the force in the $x$-direction is zero, we have

\[
\alpha_{yx} = \frac{x}{f_y}; \quad \alpha_{yy} = \frac{y}{f_y} \tag{14.13}
\]

The bearing stiffness coefficient may be obtained by inverting the influence coefficient matrix i.e. $[K] = [\alpha]^{-1}$. This method still requires a test rig which is capable of providing loads on the bearing in
both x and y directions. It is relatively easy to change the static load in vertical direction by changing the weight of the rotor. However, difficulty may occur to apply load in the horizontal direction.

**Alternative method (ii):** If there is no facility on the test rig for applying loads transverse to the normal steady-state load direction of the bearing, it is still possible to obtain approximate value of the stiffness coefficients.

In Figure 14.3, \( e_r \) is the eccentricity, \( \phi \) is the altitude angle, \( A \) is the steady state position for a vertical load \( W \), an additional imaginary static force \( x_F \) is applied in the horizontal direction to the journal to change its steady state running position to \( B \), \( R \) is the resultant force of \( W \) and \( x_F \), \( \delta \phi \) is the change in altitude angle due to additional \( x_F \), \( \delta \psi \) is the angle of \( R \) with respect to vertical direction, i.e. \( W \), and \( e_r + \delta e_r \) is the new eccentricity after application of \( x_F \). The influence coefficient can be obtained as

\[
\alpha_{x_r} = \frac{x_r}{F_x} = \frac{RB}{F_s} = \frac{PB-PR}{F_s} = \frac{PB-SA}{F_s} = \frac{(e_r + \delta e_r)\sin(\phi + \delta \phi) - e_r\sin \phi}{F_s} = \frac{(e_r + \delta e_r)(\sin \phi \cos \delta \phi + \cos \phi \sin \delta \phi) - e_r\sin \phi}{F_s}
\]

Since for small displacements, we have \( (e_r + \delta e_r) = e_r \), \( \sin \delta \phi = \delta \phi \) and \( \cos \delta \phi = 1 \). The influence coefficient can be simplified to
A further simplification can be made if the resultant $R$ is considered to be of vertically same magnitude as the original load $W$, except that it has been turned through an angle, $\delta\psi$. We may write

$$\delta\phi = \delta\psi = \tan\delta\psi = \frac{F}{W}$$  \hspace{1cm} (14.15)$$

On substituting equation (14.15) into equation (14.14), it gives

$$\alpha_{xx} = \frac{e_x \cos\phi}{F_x} \frac{F_x}{W} = \frac{e_x \cos\phi}{W} = \frac{y_0}{W}$$  \hspace{1cm} (14.16)$$

Similarly, it may be shown that

$$\alpha_{yy} = \frac{y}{F_y} = -\frac{e_y \sin\phi}{W} = \frac{u_0}{W}$$  \hspace{1cm} (14.17)$$

Since vertical load, $F_y$, is easy to apply, one can get $\alpha_{yy} = \frac{y}{F_y}$ and $\alpha_{xy} = \frac{x}{F_y}$. Then, stiffness coefficients can be obtained as $[k] = [\alpha]^T$.

**Example 14.1** Under particular operating conditions, the theoretical values of the stiffness coefficients for a hydrodynamic bearing are found to be; $k_{xx} = 30$ MN/m, $k_{xy} = 26.7$ MN/m, $k_{yx} = 0.926$ MN/m, $k_{yy} = 11.7$ MN/m. A testing is being designed so that these values can be confirmed experimentally. What increment in horizontal ($f_x$) and vertical ($f_y$) loads must the rig be capable of providing in order to provide (a) a displacement increment of 12 $\mu$m in the horizontal direction whilst that in the vertical direction is maintained zero and (b) a displacement increment of 12 $\mu$m in the vertical direction whilst that in horizontal direction is maintained zero.

**Solution:** From equation (14.7) static forces required in the $x$ and $y$ directions to a given displacement can be obtained. For case (a) following forces are required

$$f_x = 30 \times 12 = 360 \text{ N} \quad \text{and} \quad f_y = 0.926 \times 12 = 11.112 \text{ N}$$
For case (b) following forces are required

\[ f_x = 26.7 \times 12 = 320.4 \text{ N} \quad \text{and} \quad f_y = 11.7 \times 12 = 140.4 \text{ N} \]

**Answer**

**Example 14.2:** The test rig described in Example 14.1 is used to measure the hydrodynamic bearing stiffness coefficients by applying first of all a horizontal load of 360 N, which is then removed and replaced by a vertical load of 320 N. The horizontal load produces displacement of 14.3 \( \mu \text{m} \) and 3.3 \( \mu \text{m} \) in the horizontal and vertical directions respectively, whilst the vertical load produces respective displacements of \(-18.3 \mu \text{m}\) and 19.7 \( \mu \text{m} \). Calculate the value of stiffness coefficients based on these measurements.

**Solution:** For the horizontal load of 360 N alone from equation (14.10), we have

\[ \alpha_{xx} = \frac{10.3}{360} = 28.6 \times 10^{-9} \text{ m/N} \; \quad \alpha_{xy} = \frac{3.3}{360} = 9.167 \times 10^{-9} \text{ m/N} \]

For the vertical load of 320 N alone from equation (14.11), we have

\[ \alpha_{yx} = \frac{-18.3}{320} = -57.188 \text{ m/N} \; \quad \alpha_{yy} = \frac{19.7}{320} = 61.563 \text{ m/N} \]

From equation (14.9), we can obtain stiffness coefficients as

\[
\begin{bmatrix}
K
\end{bmatrix} = \begin{bmatrix}
k_{xx} & k_{yx} \\
k_{xy} & k_{yy}
\end{bmatrix} = \begin{bmatrix}
28.6 & -57.188 \\
9.167 & 61.563
\end{bmatrix}^{-1} \begin{bmatrix}
0.0269 & 0.0250 \\
-0.0040 & 0.0125
\end{bmatrix} 10^9 = \begin{bmatrix}
26.9 & 25.0 \\
-4.0 & 12.5
\end{bmatrix} \text{ MN/m}
\]

**Answer**

Myllerup *et al.* (1992) made use of the experimental static eccentricity locus (plot of the eccentricity ratio, \( \varepsilon = c_r / l_e \), versus the altitude angle, \( \phi \) and of the load-eccentricity ratio (\( W/\mu \delta \) versus \( \varepsilon \)) of the journal for fluid-film bearings. By using the locus differential method the following exact relationship of stiffness coefficients are obtained
\[
\begin{align*}
\frac{c_{k_r}}{W} &= \mu \frac{\partial}{\partial \varepsilon} \left( \frac{W}{\mu \omega} \right) \cos \phi; & \frac{c_{k_{\phi}}}{W} &= \frac{1}{\varepsilon} \cos \phi - \left( \frac{\partial \phi}{\partial \varepsilon} \right) \sin \phi; \\
\frac{c_{k_{\theta r}}}{W} &= \mu \frac{\partial}{\partial \varepsilon} \left( \frac{W}{\mu \omega} \right) \sin \phi & \frac{c_{k_{\theta \phi}}}{W} &= \frac{1}{\varepsilon} \sin \phi + \left( \frac{\partial \phi}{\partial \varepsilon} \right) \cos \phi
\end{align*}
\]

(14.18)

where \(c_r\) is the journal radial clearance, \(\mu\) is the lubricant viscosity, \(\omega\) is the journal angular speed and \(W\) is the static vertical load per bearing. While using this method, it is difficult to ensure satisfactory numerical accuracy especially at small eccentricity ratios because stiffness coefficients are calculated by differentiating the static equilibrium locus. This method does not yield damping characteristics of the bearing and it is restricted to an axi-symmetric geometry, i.e., plain cylindrical bearing.

With the assumption of a short-width journal bearing both the stiffness and damping coefficients can be obtained from the experimental static eccentricity locus using the closed form relationships (Smith, 1969; with minor typographical corrections) in the terms of the eccentricity ratio, \(e\), as

\[
\begin{align*}
\frac{c_{r\ell}}{W} &= \frac{4\pi^2 (2-\varepsilon^2) + 16\varepsilon^2}{\pi^2 (1-\varepsilon^2) + 16\varepsilon^2} \\
\frac{c_{\phi\phi}}{W} &= \frac{4\pi^2 (1-\varepsilon^2)(1+2\varepsilon^2) + 32\varepsilon^2 (1-\varepsilon^2)}{(1-\varepsilon^2)\{\pi^2 (1-\varepsilon^2) + 16\varepsilon^2\}^{3/2}} \\
\frac{c_{\phi\theta}}{W} &= -\frac{\pi \left( \pi^2 (1-\varepsilon^2)(1+2\varepsilon^2) + 32\varepsilon^2 (1+\varepsilon^2) \right) / \varepsilon (1-\varepsilon^2)^{3/2} \{\pi^2 (1-\varepsilon^2) + 16\varepsilon^2\}^{3/2}}{\pi^2 (1-\varepsilon^2)^{3/2} \{\pi^2 (1-\varepsilon^2) + 16\varepsilon^2\}^{3/2}} \\
\frac{c_{\phi\phi}}{W} &= \frac{\pi\{\pi^2 (1-\varepsilon^2)^2 - 16\varepsilon^4\}}{\varepsilon(1-\varepsilon^2)^{3/2} \{\pi^2 (1-\varepsilon^2) + 16\varepsilon^2\}^{3/2}} \\
\frac{c_{\phi\theta}}{W} &= \frac{2\pi(1-\varepsilon^2)^{3/2} \{\pi^2 (1+2\varepsilon^2) - 16\varepsilon^2\}}{\varepsilon(1-\varepsilon^2)^{3/2} \{\pi^2 (1-\varepsilon^2) + 16\varepsilon^2\}^{3/2}} \\
\frac{c_{\phi\phi}}{W} &= \frac{2\pi\varepsilon^2 (1-\varepsilon^2)^2 + 48\varepsilon^2}{\varepsilon(1-\varepsilon^2)^{3/2} \{\pi^2 (1-\varepsilon^2) + 16\varepsilon^2\}^{3/2}} \\
\frac{c_{\phi\theta}}{W} &= \frac{8\pi \{\pi^2 (1+2\varepsilon^2) - 16\varepsilon^2\}}{\varepsilon(1-\varepsilon^2)^{3/2} \{\pi^2 (1-\varepsilon^2) + 16\varepsilon^2\}^{3/2}}
\end{align*}
\]

(14.19)

or in terms of the eccentricity ratio, \(e\), and the attitude angle, \(\phi\) as given by Hamrock (1994). To the authors knowledge no experimental works of the bearing dynamic coefficients estimation have not been reported using these closed form relationships. The static load method has the advantage that it may be used on experimental steady-state locus results, thereby constituting an alternative to the analysis of dynamic response. But because of the inherent experimental difficulties, the method has not been widely adopted.
14.5. Methods Using Dynamic Loads

Most of the developments in the identification of dynamic bearing parameters have taken place in the last 55 years by using the dynamic load methods. Most authors have identified the dynamic bearing parameters by assuming a rigid rotor, although a few have allowed a flexible rotor. The excitation can be applied either to the journal or to the bearing housing (preferably floating on the journal) and it depends upon the practical constraints. The basic form of the equations of motion for rotor-bearing system remains the same irrespective of the types of excitation force used to excite the system. Now some of the forms of the equations of motion used by various researchers to develop parameter estimation methods are summarised.

For the case when the excitation is applied to the journal (Figure 14.4), the fluid-film dynamic equation, for the rigid rotor case, can be written, noting equation , as

\[
\begin{pmatrix}
    m_{xx} & m_{xy} \\
    m_{yx} & m_{yy}
\end{pmatrix}
\begin{pmatrix}
    \ddot{x} \\
    \ddot{y}
\end{pmatrix}
+ \begin{pmatrix}
    c_{xx} & c_{xy} \\
    c_{yx} & c_{yy}
\end{pmatrix}
\begin{pmatrix}
    \dot{x} \\
    \dot{y}
\end{pmatrix}
+ \begin{pmatrix}
    k_{xx} & k_{xy} \\
    k_{yx} & k_{yy}
\end{pmatrix}
\begin{pmatrix}
    x \\
    y
\end{pmatrix}
= \begin{pmatrix}
    f_x - m(\ddot{x} + \dot{x}_b) \\
    f_y - m(\ddot{y} + \dot{y}_b)
\end{pmatrix}
\]

(14.20)

where \(m\) is the mass of the journal and \(x\) and \(y\) represent the motion of the journal center from its equilibrium position relative to the bearing center and \(x_b\) and \(y_b\) are the components of the absolute displacement of the bearing center in vertical and horizontal directions respectively. It is assumed here that the origin of the co-ordinate system is at the static equilibrium position, so that the gravity force does not appear in the equation of motion. The majority of researchers have ignored the added-mass effects of the fluid-film, the bearing is considered as fixed in space (i.e., \(x_b = y_b = 0\); see Figure 14.5) and bearing dynamic coefficients are assumed to be excitation frequency, \(\Omega\), independent (however, with the rotational frequency of the rotor, \(\omega\) dependent). Equation (14.20) may be further simplified by assuming the inertia force due the journal motion as negligible for lower excitation frequencies.

![Figure 14.4 A non-floating bearing housing and a rotating journal floating on the fluid](image-url)
When the dynamic force is applied to the bearing housing (preferably the floating bearing on the rigidly mounted rotating shaft, see Figure 14.6), the equation of motion of the bearing housing may be written as

\[
\begin{pmatrix}
 m_{xx} & m_{xy} \\
 m_{yx} & m_{yy}
\end{pmatrix}\begin{pmatrix}
 \ddot{x} \\
 \ddot{y}
\end{pmatrix} + \begin{pmatrix}
 c_{xx} & c_{xy} \\
 c_{yx} & c_{yy}
\end{pmatrix}\begin{pmatrix}
 \dot{x} \\
 \dot{y}
\end{pmatrix} + \begin{pmatrix}
 k_{xx} & k_{xy} \\
 k_{yx} & k_{yy}
\end{pmatrix}\begin{pmatrix}
 x \\
 y
\end{pmatrix} = \begin{pmatrix}
 f_x - m_b \ddot{x}_b \\
 f_y - m_b \ddot{y}_b
\end{pmatrix}
\]  

\[(14.21)\]

where \( m_b \) is the mass of the bearing and its associated structure, \( x \) and \( y \) are the components of the dynamic displacement of the bearing centre relative to the journal centre and \( x_b \) and \( y_b \) are the components of the absolute displacement of the bearing centre.
Figure 14.7 A schematic diagram of the flexible rotor-bearings system

For the case when the flexibility of the rotor, along with gyroscopic, rotary inertia and shear effects are also considered, the governing equation of multi-degree-of-freedom (MDOF) rotor-bearing system (see Figure 14.7) for transverse vibrations in general form may be written as

\[
[M]{\ddot{\eta}} + ([C] - \omega[G])[\dot{\eta}] + ([K] + [H])\{\eta\} = \{f(t)\}
\]

and

\[
\{\eta(t)\} = \{\eta_1(t) \quad \eta_2(t) \quad \cdots \quad \eta_n(t)\}^T; \quad \{\eta_i(t)\} = \{x_i(t) \quad \varphi_{x_i}(t) \quad y_i(t) \quad \varphi_{y_i}(t)\}^T;
\]

\[
\{f(t)\} = \{f_1(t) \quad f_2(t) \quad \cdots \quad f_n(t)\}^T; \quad \{f_i(t)\} = \{f_{x_i}(t) \quad M_{x_i}(t) \quad f_{y_i}(t) \quad M_{y_i}(t)\}^T.
\]

(14.22)

where the positive definite, but not necessarily diagonal, matrix \([M]\) is known as the mass (inertia) matrix, the skew-symmetric matrices \([G]\) and \([H]\) are referred to as the gyroscopic and the circulatory matrices respectively, the indefinite non-symmetric matrices \([C]\) and \([K]\) are called the damping and the stiffness matrices respectively, \(x\) and \(y\) are the linear displacements, \(\varphi_x\) and \(\varphi_y\) are the angular displacements, \(f_x\) and \(f_y\) are forces and, \(M_{x_i}\) and \(M_{y_i}\) are moment vectors. The matrices \([M]\), \([G]\), \([C]\), \([K]\) and \([H]\) are in general rotational speed, \(\omega\) dependent and for a given \(\omega\) equation (14.22) can be written as

\[
[M]{\ddot{\eta}(t)} + [C][\dot{\eta}(t)] + [K][\eta(t)] = \{f(t)\}
\]

(14.23)

with

\[
[C] = [C] - \omega[G] \quad \text{and} \quad [K] = [K] + [H]
\]
where the generalised damping and stiffness matrices, $[C]$ and $[K]$, are now neither positive (negative) definite nor symmetric. Different forms of equation (14.23) have been used, in general, for obtaining the bearing parameter estimation equations. Equation (14.23) can be written as

$$
[M_r][\ddot{\eta}] + ([C_r]+[C_b])[\dot{\eta}] + ([K_r]+[K_b])[\eta] = \{f(t)\}
$$

(14.24)

where subscripts: $r$ and $b$ represent for the rotor and bearing respectively. This form was used by Arumugam et al. (1995) and Chen and Lee (1995, 1997) to extract $[K_b]$ and $[C_b]$ in terms of the known and measurable quantities such as the rotor model, forcing and corresponding response.

Equation (14.23) can also be written in frequency domain for MDOF rotor-bearing systems, as

$$
[Z]\{Q\} = \{F_u\} + j\omega[C]
$$

(14.25)

with

$$
[Z] = ([K] - j\omega^2[M]) + j\omega[C]
$$

where $[Z]$ is the dynamic stiffness matrix, $\{F_u\}$ is the unbalance force, $\omega$ is the rotational frequency of the rotor, the first subscripts, $R$ and $B$, refer to rotor and bearing respectively, and the second subscripts, $i$ and $b$, correspond to internal and connection DOFs, respectively. The DOFs of the rotor at the bearing locations are called connection DOFs, $\{Q_{R,b}\}$, and the DOFs of the rotor other than at the bearing locations are called internal DOFs, $\{Q_{R,i}\}$. It is assumed here that balance planes (unbalances) are present only at the rotor internal DOFs. Tiwari et al. (2002) used equation (14.25) to extract the bearing parameters, $[Z_{B,bb}]$, in terms of the known and measurable quantities. For bearing dynamic parameter identification in the multi-DOF rotor-bearing systems, the finite element formulation is quite popular and a very few researchers attempted by using the transfer matrix method (Lee et al., 1993).

The basic form of the estimation equation is the same for both the rigid and flexible rotor cases. For the most general case, when the bearing dynamic coefficients depend upon both the frequency of external excitation, $\Omega$, and the rotational frequency of the rotor, $\omega$. Effectively, there are four equations for the eight unknown bearing parameters (for example for hydrodynamic bearings effective stiffness and damping coefficients and for squeeze-film bearings added-mass and damping coefficients). To estimate all eight dynamic coefficients, the response corresponding to at least two
independent sets of force (only in the magnitude &/or phase but not in the frequency, since bearing dynamic parameters could be external excitation frequency dependent) is required. Even for MDOF rotor-bearing system the response at each bearing locations corresponding to at least two independent sets of force are required to find eight dynamic coefficients of all the bearings in the system (Tiwari et al., 2002). For the case when bearing dynamic coefficients are rotor rotational frequency dependent but not the external excitation frequency dependent, all twelve dynamic coefficients (i.e. added-mass, damping and stiffness) can be estimated, in principle, by using force-response measurements at the minimum of three excitation frequencies (magnitude and phase may or may not be same). All other methods except method using the unbalance (run-downs/ups) would be able to satisfy above condition. While using the unbalance response where the external excitation frequency is equal to the frequency of rotation of the rotor, by changing the rotor rotational frequency (i.e. the frequency of excitation) would result in change in bearing dynamic parameters. So it is not possible to obtain all twelve bearing dynamic coefficients by using synchronous unbalance response (run-downs/ups) and only eight bearing dynamic coefficients (i.e. damping and stiffness) could be estimated. For the case when the bearing dynamic parameters are independent of both the frequency of excitation and the rotational frequency of the rotor all twelve coefficients could be obtained by any method by using the force-response data corresponding to the minimum of three frequency points. Finally, it is unlikely that bearing dynamic parameters depend upon external excitation frequency but do not depend upon rotational frequency of the rotor.

14.6 Development of a General Identification Algorithm in Rotor-Bearing Systems

The linearised model for any rotating machine (assuming no internal rotor damping) has the form

$$
M\ddot{q} + [C + \omega_1 G_1 + \omega_2 G_2 + \cdots] \ddot{q} + Kq = f
$$

(14.26)

where $M$ is the mass matrix, $K$ is the stiffness matrix, $C$ is the damping matrix and $G_1$, $G_2$ etc. are gyroscopic matrices (skew-symmetric) associated with the different parts of the "rotational system" spinning at the different speeds $\omega_1$, $\omega_2$, etc. Transforming equation (14.26) into the frequency domain produces this form

$$
D(\Omega) p(\Omega) = \left[ \Omega^2 M + j\Omega [C + \omega_1 G_1 + \omega_2 G_2 + \cdots] + K \right] p(\Omega) = g(\Omega)
$$

(14.27)

where $g(\Omega)$ is non-zero only for the discrete frequencies $\Omega = \omega_1$, $\Omega = \omega_2$, $\Omega = \omega_3$, etc. (assuming that the only forcing on the system is due to rotating unbalances). Here $D(\Omega)$ is the dynamic stiffness
matrix of the complete rotating machine system at forcing frequency $\omega$. The general form of the $g(\Omega)$ may be written

$$g(\Omega_i) = \Omega^2 S_i u_i$$  \hspace{1cm} (14.28)$$

where matrices $S_1$, $S_2$, $S_3$ etc. are "selection" matrices which reflect not only that the unbalance at certain different speeds can exist only at different "shaft-stations" but they also contain the information that for positive $\Omega$, the forcing in the $y$-direction lags forcing in the $x$-direction by 90 degrees.

Now, partition $p$ into a part which is measured ($p_m$) and a part which is not measured ($p_u$).

$$p = \begin{bmatrix} p_m \\ p_u \end{bmatrix}$$  \hspace{1cm} (14.29)$$

and partition $D(\Omega)$ accordingly thus

$$D(\Omega) = \begin{bmatrix} D_{mm}(\Omega) & D_{ms}(\Omega) \\ D_{sm}(\Omega) & D_{ss}(\Omega) \end{bmatrix}$$  \hspace{1cm} (14.30)$$

Note that $D$ itself contains some parts which are known with good confidence and some parts which are quite unknown (the bearing parameters would be in this category). In order to avoid confusion with subscripts, these two separate contributions to $D$ are denoted $E$ and $F$ respectively where $E$ is the known part and $F$ is the unknown part.

$$D(\Omega) = E(\Omega) + F(\Omega)$$  \hspace{1cm} (14.31)$$

Matrices $E$ and $F$ are partitioned in the same way as $D$. Now combine equations (14.27) to (14.31).

$$\begin{bmatrix} F_{mm}(\Omega_i) & F_{mu}(\Omega_i) \\ F_{um}(\Omega_i) & F_{uu}(\Omega_i) \end{bmatrix} \begin{bmatrix} p_m(\Omega_i) \\ p_u(\Omega_i) \end{bmatrix} = \Omega^2 S_i u_i = \Omega^2 \begin{bmatrix} S_{im} \\ S_{iu} \end{bmatrix} u_i$$  \hspace{1cm} (14.32)$$

Observe that the $S_i$ are known. Now, in those cases where $F_{mu}$ and $F_{uu}$ are both zero, equation (14.33) can clearly be set out as a linear relationship between the known quantities and the unknowns. The
detail of how the unknown quantities are arranged in a vector is irrelevant. The ordering neither helps nor hinders the subsequent processing of this information. After choosing an appropriate ordering, the problem manifests itself as

$$Ax = b$$  \hspace{1cm} (14.33)$$

where \(x\) is the vector of all unknowns (to be determined) and \(b\) is a vector of known quantities. In subsequent sections the issue of what to do to make sure that \(A\) is well-conditioned will be discussed in detail. Now various methods of estimation of bearing parameters based on dynamic forces are discussed.

### 14.7 Use of Electromagnetic Vibrators

In order to fully analyse the behaviour of a bearing under dynamic loading it is necessary to cause the journal to vibrate within the bearing bush under the action of a known exciting force as shown in Figure 14.6. In practical situations the application of a calibrated force to the journal is not always feasible. Alternatively, the bearing bush can be allowed to float freely on the journal as shown in Figure 14.7, which is mounted on a slave bearings and the forcing is applied to the bush. By measuring the resulting system vibrations and relating these to the force, it is possible to determine the effective oil-film stiffness and damping coefficient. By varying the amplitude, frequency and shape of the electrical signal input to the vibrator it is possible to exercise full control over the forcing applied to the system.

#### 14.7.1 Complex Receptance Method

The method involves applying a sinusoidally varying force to the journal in the horizontal direction, whilst the forcing in the vertical direction is zero, and measuring the resulting displacement amplitudes in the horizontal and vertical directions together with their respective phase relative to the exciting force. It is then necessary to repeat the procedure with the forcing applied only in the vertical direction. The knowledge of force amplitude and measured displacement quantities, then enables the eight oil-film coefficient to be derived. The force transmitted across the oil-film may be represented in the form

$$f_x = k_{xx}x + k_{xy}y + c_{xx}\dot{x} + c_{xy}\dot{y} \quad \text{and} \quad f_y = k_{yx}x + k_{yy}y + c_{yx}\dot{x} + c_{yy}\dot{y}$$  \hspace{1cm} (14.34)$$

Assuming sinusoidal variations of \(x\) and \(y\) (i.e. \(x = X e^{j\omega t}\) etc., where \(\omega\) is frequency of forcing function), equation (14.12) gives
\[ f_x = (k_{xx} + j\omega c_{xx})x + (k_{xy} + j\omega c_{xy})y \quad \text{and} \quad f_y = (k_{yx} + j\omega c_{yx})x + (k_{yy} + j\omega c_{yy})y \] (14.35)

which can be written in matrix form as

\[
\begin{bmatrix}
  f_x \\
  f_y
\end{bmatrix} =
\begin{bmatrix}
  Z_{xx} & Z_{xy} \\
  Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\] (14.36)

where \( Z \) is a complex stiffness coefficient given by \( Z = k + j\omega \)

\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix} =
\begin{bmatrix}
  R_{xx} & R_{xy} \\
  R_{yx} & R_{yy}
\end{bmatrix}
\begin{bmatrix}
  f_x \\
  f_y
\end{bmatrix}
\] (14.37)

where \([R] = [Z]^{-1}\) is called the complex receptance matrix (also termed the dynamic compliance (DC), frequency response function (FRF) and transfer function (TF)). It is defined as the complex quotient of a displacement and force vector. The indices of the dynamic stiffness coefficients and receptances have the similar significance to that for the bearing dynamic coefficients and influence coefficients, respectively. For the case of forcing in horizontal direction only equation (14.37) gives

\[ R_{xx} = \frac{x}{f_x} \quad \text{and} \quad R_{yy} = \frac{y}{f_y} \] (14.38)

where \( x \) and \( y \) are the measured displacement in the horizontal and vertical directions at a particular time and \( f_x \) is the force in the horizontal direction at that instant. For the case when forcing is in the vertical direction the other reacceptance terms are derived as

\[ R_{yx} = \frac{x}{f_y} \quad \text{and} \quad R_{yy} = \frac{y}{f_y} \] (14.39)

On inverting \([R]\) elements of \([Z]\) can be obtained. The elements of \([Z]\) contain all eight bearing stiffness and damping coefficients as defined in equation (14.36) i.e. \([Z] = [K] + j\omega[C]\) where \( \omega \) is the frequency of forcing function. Problems related to this method can be easily solved in the MATLAB and is illustrated in following examples.
Example 14.3 A bearing is forced in the horizontal direction by a force $F_x = 150 \sin 200t$ N. The resulting vibrations are $x = 7 \times 10^{-6} \sin(200t - 0.2)$ m in the horizontal direction and $y = 20 \times 10^{-6} \sin(200t - 0.32)$ m in the vertical direction. When the same forcing is applied in the vertical direction the horizontal and vertical displacements take the respective forms $x = 8 \times 10^{-6} \sin(200t + 0.15)$ m and $y = 26 \times 10^{-6} \sin(200t - 0.3)$ m. Determine dynamic coefficients of the bearing.

Solution: We have two sets of measurements

(i) For $f_x(t) = 150 \sin 200t$ N and $f_y(t) = 0$

$x = 7 \times 10^{-6} \sin(200t - 0.2)$ m and $y = 20 \times 10^{-6} \sin(200t - 0.32)$ m

which can be written in complex plane as

For $f_x(t) = 150e^{200t} \cos(200t)$ alone, we have

\[ x = 7 \times 10^{-6} e^{(200t-0.2)} \] \[ y = 20 \times 10^{-6} e^{(200t-0.32)} \] \[ \text{(a)} \]

and

(ii) $f_x(t) = 0$ and $f_y(t) = 150 \sin 200t$ N

$x = 0.8 \times 10^{-6} \sin(200t + 0.15)$ m and $y = 26 \times 10^{-6} \sin(200t - 0.3)$ m

which can be written in complex plane as

For $f_y(t) = 150e^{200t} \cos(200t)$ alone, we have

\[ x = 0.8 \times 10^{-6} e^{(200t+0.15)} \] \[ y = 26 \times 10^{-6} e^{(200t-0.3)} \] \[ \text{(b)} \]

Bearing dynamic coefficients are defined as

\[ f_x(t) = k_{xx}x + k_{xy}y + c_{xx}\dot{x} + c_{xy}\dot{y} \] \[ f_y(t) = k_{yx}x + k_{yy}y + c_{yx}\dot{x} + c_{yy}\dot{y} \] \[ \text{(c)} \]

On substituting the first set of measurement from equation (a) into equation (c), we have
\[
\begin{pmatrix}
150e^{j200r} \\
0
\end{pmatrix} = \begin{pmatrix}
(k_{xx} + j200c_{xx}) & (k_{yx} + j200c_{yx}) \\
(k_{yy} + j200c_{yy}) & (k_{yy} + j200c_{yy})
\end{pmatrix} \begin{pmatrix}
150e^{j200r} \\
0
\end{pmatrix}
\]

Similarly on substituting the second set of measurement from equation (b) into equation (c), we have

\[
\begin{pmatrix}
0 \\
150e^{j200r}
\end{pmatrix} = \begin{pmatrix}
(k_{xx} + j200c_{xx}) & (k_{yx} + j200c_{yx}) \\
(k_{yy} + j200c_{yy}) & (k_{yy} + j200c_{yy})
\end{pmatrix} \begin{pmatrix}
8 \times 10^{-6} e^{j(200+0.15)r} \\
26 \times 10^{-6} e^{j(200-0.3)r}
\end{pmatrix}
\]

Let the dynamic stiffness is defined as

\[
Z = k + j\omega c \quad \text{with} \quad \omega = 200 \text{ rad/sec}
\]

First set of equations from equations (d) and (e), we have

\[
7 \times 10^{-6} e^{j(200-0.2)r} Z_{xx} + 20 \times 10^{-6} e^{j(200-0.3)r} Z_{yy} = 150e^{j200r}
\]

and

\[
8 \times 10^{-6} e^{j(200-0.15)r} Z_{xx} + 26 \times 10^{-6} e^{j(200-0.3)r} Z_{yy} = 0
\]

Equation (g) gives

\[
Z_{xx} = -\frac{26 \times 10^{-6} e^{j(200-0.3)r}}{8 \times 10^{-6} e^{j(200-0.3)r}} Z_{yy} \quad \text{or} \quad Z_{xx} = -3.25e^{-0.45} Z_{yy}
\]

On substituting equation (h) into equation (f), we get

\[
Z_{xx} = \frac{150\epsilon^{j200r}}{26 \times 10^{-6} e^{j(200-0.3)r} - 22.75 \times 10^{-6} e^{j(200-0.2)r}} = \frac{150}{20 \times 10^{-6} e^{-0.32} - 22.75 \times 10^{-6} e^{-0.2j}}
\]

or

\[
Z_{yy} = -3.52 \times 10^7 + j1.884 \times 10^7
\]

On substituting equation (i) into equation (h), we get

\[
Z_{yy} = 7.64 \times 10^7 - j1.049 \times 10^8
\]
Similarly from first set of equation (e), we have

\[
Z_{yx} = -Z_{yx} \left( \frac{20 \times 10^{-6} e^{j(200t - 0.32)}}{7 \times 10^{-6} e^{j(200t - 0.2)}} \right) \Rightarrow Z_{yx} = -2.8571 e^{-0.12j} Z_{yy}
\]  

(k)

On substituting equation (k) into second equation of (e), we get

\[
Z_{yy} = \frac{150 e^{200t}}{26 \times 10^{-6} e^{j(200t - 0.3)} - 22.86 \times 10^{-6} e^{j(200t + 0.15)}} = \frac{150 \times 106}{26 \times e^{-0.3} - 22.86 \times e^{j0.15}}
\]  

(l)

In simplification of equations (k) and (l), we get

\[
Z_{yx} = 2.6156 \times 10^6 + j 1.2987 \times 10^7 \quad \text{and} \quad Z_{yx} = -1.186 \times 10^7 + j 3.594 \times 10^7
\]  

(m)

Stiffness and damping coefficients can be obtained by separating real and imaginary part of the dynamic stiffness coefficients from equations (i), (j) and (m), as

\[
k_{xx} = 7.64 \times 10^7 \text{ N/m}; \quad k_{yy} = -3.52 \times 10^7 \text{ N/m}; \quad k_{xy} = -1.186 \times 10^7 \text{ N/m}; \quad k_{yx} = 2.6156 \times 10^6 \text{ N/m}
\]

\[
c_{xx} = -524500 \text{ N/m-sec}; \quad c_{xy} = 94200 \text{ N/m-sec}; \quad c_{yx} = -179700 \text{ N/m-sec}; \quad c_{yy} = 64935 \text{ N/m-sec}
\]

**Answer**

**Example 14.4** A bearing is forced in the horizontal direction by a force \( F_x = 150 \sin 200t \text{ N} \). The resulting vibrations are \( x = 7 \times 10^{-6} \sin (200t - 0.2) \) meters in the horizontal direction and \( y = 20 \times 10^{-6} \sin (200t - 0.32) \) meters in the vertical direction. When the same forcing is applied in the vertical direction, the horizontal and vertical displacements take the respective forms \( x = 8 \times 10^{-6} \sin (200t + 0.15) \) meters and \( y = 26 \times 10^{-6} \sin (200t - 0.3) \) meters. Determine elements of complex receptance matrix for the bearing.

**Solution**: The following measurement were done

Case I: For \( F_x = 150 \sin 200t \text{ N} \) and \( F_y = 0 \), we have

\[
x = 7 \times 10^{-6} \sin (200t - 0.2) \text{ m} \quad \text{and} \quad y = 20 \times 10^{-6} \sin (200t - 0.32) \text{ m}
\]
Case II: For \( F_y = 150 \sin 200t \) N and \( F_x = 0 \), we have

\[ x = 8 \times 10^{-6} \sin(200t + 0.15) \text{ m and } y = 26 \times 10^{-6} \sin(200t - 0.3) \text{ m} \]

For a force \( F_x \) leading a displacement \( X \) by \( \theta \) is shown in Figure 14.8.

![Figure 14.8 The phase between the displacement and force vectors](image)

From Figure 14.8, the receptance can be expressed as

\[ R_{xx} = \frac{X}{F_x \cos \theta + j F_x \sin \theta} \]

where \( X \) and \( F_x \) are displacement and force amplitudes. The displacement is lagging behind force by \( \theta \) angle.

![Figure 14.9 A typical force and displacement vectors](image)

From Figure 14.9, we have

\[ R_{xx} = \frac{x}{F_x} = \frac{X}{F_x \cos \theta + j F_x \sin \theta} = \frac{7 \times 10^{-6}}{150 \cos 0.2 + j 150 \sin 0.2} = (0.04574 - j 0.00927) \times 10^{-6} \text{ m/N} \]

\[ R_{yy} = \frac{y}{f_x} = \frac{26 \times 10^{-6}}{150 \cos(0.32) + j 150 \sin(0.32)} = (0.1266 - j 0.042) \times 10^{-6} \text{ m/N} \]
\[ R_{yy} = \frac{y}{f_y} = \frac{26 \times 10^{-6}}{150 \cos(0.3) + j 150 \sin(0.3)} = (0.046 + j 0.007) \times 10^{-6} \text{m/N} \]

\[ R_{xy} = \frac{x}{f_y} = \frac{8 \times 10^{-6}}{150 \cos(0.15) + j 150 \sin(0.15)} = (0.127 - j 0.0394) \times 10^{-6} \text{m/N} \]

Hence, the receptance matrix can be written as

\[ [R] = \begin{bmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{bmatrix} = \begin{bmatrix} (45.74 - j 9.27) & (52.73 + j 7.97) \\ (126.56 - j 41.94) & (165.59 + j 51.22) \end{bmatrix} \frac{\text{m}}{\text{MN}} \]

**Answer**

### 14.7.2 Estimation using the Direct Complex Impedance

It is possible to determine the complex stiffness coefficients \( Z_{xx}, Z_{yy} \) etc. in equation (14.36) directly without resorting to the use of receptances. This may be done provided that forcing in both the horizontal and vertical directions be able provide simultaneously, and with independent control over each input with respect to its amplitude and relative phase. The system force-displacement relationship is given by equation (14.36).

**Method 1:** In the present approach it is to ensure that one of the resulting system displacement vectors, say \( y \), is zero. This can be made to be the case by correctly adjusting the amplitude of the force in the \( y \)-direction and its phase relative to that in the \( x \)-direction. Suitable values for these quantities can be found relatively easily by trial and error. The first line of equation (14.36) thus gives

\[ Z_{xx} = \frac{f_y}{x} \]

which will allow the value of \( Z_{xx} \) to be determined directly provided that the amplitude and phase of the horizontal displacement relative to the horizontal force had been measured. If the amplitude and phase of the force in the \( y \)-direction were also noted then the value of \( Z_{yy} \) could also be determined as

\[ Z_{yy} = \frac{f_y}{x} \]

In the above case, it is the phase of the \( x \)-direction displacement amplitude relative to the force in the \( y \)-direction that is significant. Similarly by adjusting forcing amplitudes and relative phases so as to ensure a zero horizontal displacement, \( x \), then the values of \( Z_{xy} \) and \( Z_{yx} \) could also be determined.
This method requires more complicated experimental procedure. It may be more costly in terms of equipment, since two vibrators and additional control units are required.

Method 2: Parkins (1979, 1981 and 1995) used a slightly different method by applying oscillatory vertical and horizontal forces to the journal and whose relative phase and magnitude may be independently adjusted so that either

(a) \( x = \dot{x} = 0 \) at all time \( t \). Then equation (14.35) gives

\[
\begin{align*}
    k_{xy} &= F_x / y \quad \text{and} \quad k_{yx} = F_y / y \quad \text{for} \quad \dot{y} = 0, \quad y \neq 0 \\
    c_{xy} &= F_x / \ddot{y} \quad \text{and} \quad c_{yx} = F_y / \ddot{y} \quad \text{for} \quad \dot{y} \neq 0, \quad y = 0
\end{align*}
\]  

(14.42)

or

(b) \( y = \dot{y} = 0 \) at all time \( t \). Then

\[
\begin{align*}
    k_{xx} &= F_x / x \quad \text{and} \quad k_{yx} = F_y / x \quad \text{for} \quad \dot{x} = 0, \quad x \neq 0 \\
    c_{xx} &= F_x / \ddot{x} \quad \text{and} \quad c_{yx} = F_y / \ddot{x} \quad \text{for} \quad \dot{x} \neq 0, \quad x = 0
\end{align*}
\]  

(14.43)

This is identical to shaking the journal in vertical and horizontal straight-line harmonic motions. Each of the eight coefficients is computed from a single equation, which utilises only two measured quantities. This method eliminates the inversion of a dynamic stiffness matrix with no measurement of phase between journal displacement (or velocity) and applied force. But the experimentation is quite involved and difficult under operating conditions because it needs adjustment of the amplitude and phase of two exciters simultaneously to get the response in one or other direction equal to zero. It was concluded that for higher eccentricity ratios (greater than 0.78) the nonlinearity in the measured coefficients nonlinearly was found to be significant. Parkins (1981) expressed the bearing dynamic coefficients in terms of a constant value and a linear gradient as

\[
\begin{align*}
    k_{xx} &= k_{xx0} + \alpha_{xx} x, \\
    c_{xx} &= c_{xx0} + \beta_{xx} y
\end{align*}
\]  

(14.44)

where \( k_{xx0} \) and \( c_{xx0} \) represent the intercept with the zero displacement or velocity axis and are called the zero coefficients, \( \alpha \) and \( \beta \) represent gradients and \( x \) and \( y \) are the vertical and horizontal journal center co-ordinates from the static equilibrium position, respectively. He proposed the selected orbit technique wherein the coefficients were obtained from special orbits with straight lines at the
crossover point. Subsequently in 1995 he extended the method so that production of a figure eight shaped orbit was required and the crossover point was utilised for estimation of bearing parameters. The orbit was obtained by the application of mutually perpendicular forces, whose relative magnitude and phase could be adjusted and whose exciting frequencies were $\Omega$ and $2\Omega$. Jing et al. (1998) described an on-line procedure for measuring the four damping coefficients of fluid-film journal bearings from imposed dynamic orbits of a figure of eight. A microcomputer was used to control the dynamic forces, detect the figure of eight shaped orbit, find the crossover point and finally, compute the damping coefficients.

Method 3: Someya (1976), Hisa et al. (1980) and Sakakida et al. (1992) identified the dynamic coefficients of large-scale journal bearing by using simultaneous sinusoidal excitations on the bearing at two different frequencies and measuring the corresponding displacement responses. This method is called bi-directional compound sinusoidal excitation method and all the eight bearing dynamic coefficients could be obtained by single test measurements. When the journal vibrates around the static position of equilibrium in a bearing, the dynamic component of the reaction force of the fluid-film can be expressed by equation (14.34). If the excitation force and dynamic displacement are measured at two different excitation frequencies under the same static state of equilibrium then equation (14.34), ignoring the fluid-film added-mass effects, can be solved for the eight unknown coefficients as

\[
\begin{bmatrix}
X_1 & Y_1 & j\Omega X_1 & j\Omega Y_1 \\
X_2 & Y_2 & j\Omega X_2 & j\Omega Y_2
\end{bmatrix}
\begin{bmatrix}
k_{xx} \\
k_{xy}
\end{bmatrix}
= \begin{bmatrix}
F_{x_1} - m_b\Omega^2 X_1 \\
F_{x_2} - m_b\Omega^2 X_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_1 & Y_1 & j\Omega X_1 & j\Omega Y_1 \\
X_2 & Y_2 & j\Omega X_2 & j\Omega Y_2
\end{bmatrix}
\begin{bmatrix}
k_{yx} \\
k_{yy}
\end{bmatrix}
= \begin{bmatrix}
F_{y_1} - m_b\Omega^2 Y_1 \\
F_{y_2} - m_b\Omega^2 Y_2
\end{bmatrix}
\]

(14.45)

where $m_b$ is the bearing mass, $\Omega$ is the external excitation frequency and the subscripts 1 and 2 represent the measurements corresponding to two different excitation frequencies. Since equation (14.45) corresponds to effectively eight real equations, on substituting the measured values of the complex quantities $F_x, F_y, X, Y, X_b$ and $Y_b$, the bearing dynamic coefficients can be obtained. Similar procedure for all twelve dynamic parameters of seals the following estimation equations can be developed.
where the subscripts 1, 2 and 3 represent the measurements corresponding to three different excitation frequencies. Equations (14.46) and (14.47) can be used for more number of excitation frequencies to get a better estimate of the seal parameters.

Some Practical Issues: Some experimental measurement considerations are discussed now. Choice of forcing frequency (or forcing frequency range) is an important parameter to choose. It depends upon the system resonance. If the system is excited close to its resonant frequency then a response of suitable magnitude may be obtained for a lower force amplitude input. (since the bearing impedance changes with journal vibration non-linearity effect will play a major roll). The advantage of exciting the system at a frequency in the region of its resonant frequency that is that phase lag will be generally greater than zero. (between the response and the force). With this for small inaccuracies in their (phase) measurement are less likely to substantially alter the magnitude of coefficients, which are derived. This is not the case hen the lag angle is very small or when it is close to 90°. In these cases (i.e. close to 0° and 90°) ‘ill conditioning’ of the equation of motion results in significant changes in the magnitude of the derived coefficient for even a change of only 3-4 degrees of phase (which may be about the accuracy to which phase is measured). This coefficient derived using data generated well away from the critical speed may well be considerably in accurate (of the order of 100% in some cases). Also since at critical speeds it is observed that the orbit of the shaft centre is elliptical in nature and that leads to well conditioning of regression matrix.
14.7.3 Estimation by Multi-Frequency Testing

It has advantage that the certainty of exciting all system modes within the prescribe frequency range, and inherent high noise rejection. The method involves forcing the system in both $x$ and $y$ directions, at all frequencies within the range of interest, simultaneously. The aim is to arrive at more accurate values of the coefficients, which are assumed to be independent of frequency, by means of some averaging procedure. When all (several) frequencies are excited simultaneously, the knowledge of bearing behavior at many different frequencies should enable more accurate results to be obtained. Also it saves the laboratory time. Fourier analysis can be used to convert measured input and output signals from the time domain to the frequency domain. Recent advances in laboratory instrumentation, for example, the emergence of spectrum analysers (FFT analysis) capable of carrying out the Fourier transform, have helped the technique to evolve. In theory, any shape of input signal with multi-frequency content can be used to force the system. For example an impulse signal (Figure 14.10) is actually composed of signals at all frequencies in coexistence. Because of the likely concentration of the signal at the low-frequency end of the spectrum however, a impulse in practice provides useful signals over only a relatively small frequency range. For higher frequency the signal/noise ratio becomes too low. An alternative is a white noise signal, which contains all frequencies within its spectrum Band-limited white noise, sometimes referred to as coloured noise, contains all frequencies within a prescribed range. One way of producing such a signal is with ‘pseudo random binary sequences’ (PRBS) where the frequency range that is present is chosen to excite appropriate modes in the system under test. Unfortunately, both with impulse and PRBS signals there is a danger of saturating the system so that amplitudes at some frequencies are so large that non-linearity are encountered and the test becomes invalid. These disadvantages can be overcome by using a signal mode up of equal-amplitude sinusoidal signals whose frequency are those which one wishes to excite within a particular frequency range.

![Figure 14.10 Impulse and white noise signals in the time and frequency domains](image-url)
One signal of this type is ‘Schroeder phased harmonics’ (SPHS) (Schroeder, 1970). The basic attraction of SPHS is that they are synthesised from harmonic components, thus an arbitrarily specified signal spectrum can be generated with any desired frequency resolution. This flexibility in selecting the signal characteristics is desired to suppress the excitation of certain frequencies in a complex system. Using a digital computer the SPHS signal can be generated with a flat spectrum and having sharp cut-off.

If the system response to a multi-frequency signal is recorded, bearing properties may be obtained as follows. The displacement in the $x$ and $y$ directions occurring at a frequency $\omega$ are written in the form

$$x = X e^{i\omega t} \quad \text{and} \quad y = Y e^{i\omega t}$$

(14.48)

Thus

$$\dot{x} = j\omega X e^{i\omega t}; \quad \dot{y} = j\omega Y e^{i\omega t}; \quad \ddot{x} = -\omega^2 X e^{i\omega t} \quad \text{and} \quad \ddot{y} = -\omega^2 Y e^{i\omega t}$$

(14.49)

The forcing function may similarly be defined as

$$f_x = F_x e^{i\omega t} \quad \text{and} \quad f_y = F_y e^{i\omega t}$$

(14.50)

Equations of motion of the journal in the $x$ and $y$ directions are

$$f_x - k_{xx} x - k_{xy} y - c_{xx} \dot{x} - c_{xy} \dot{y} = m \ddot{x} \quad \text{and} \quad f_y - k_{yx} x - k_{yy} y - c_{yx} \dot{x} - c_{yy} \dot{y} = m \ddot{y}$$

(14.51)

where $M$ is the mass of journal; and $k_{xx}, c_{xx}$ etc. are oil film stiffness and damping coefficients. Substituting equations (14.48) to (14.50) into equation (14.51), we get

$$\begin{bmatrix} Z_{xx} (\omega) & Z_{xy} (\omega) \\ Z_{yx} (\omega) & Z_{yy} (\omega) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} F_x + m\omega^2 X \\ F_y + m\omega^2 Y \end{bmatrix}$$

(14.52)

where $Z = k + j\omega c$. On separating the real and imaginary parts, we rearrange equation (14.52) as
Equation (14.53) may be written for \( \omega = \omega_0, 2\omega_0, 3\omega_0, \ldots, n\omega_0 \) (total of \( n \) times in all). Values of \( \omega \) and quantities in the first and last matrices of equation (14.51) are determined by the Fourier transformation of time-domain signals. All of these equations (14.53) may be grouped as a single matrix equation as

\[
\begin{bmatrix}
X' & \omega X' & Y' & \omega Y' & -F'_x & -F'_y \\
X' & -\omega X' & Y' & -\omega Y' & -F'_x & -F'_y
\end{bmatrix} = \begin{bmatrix}
k_{xx} & k_{yx} \\
c_{xx} & c_{yx} \\
k_{xy} & k_{yy} \\
c_{xy} & c_{yy} \\
1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
m\omega^2 X' & m\omega^2 Y' \\
m\omega^2 X' & m\omega^2 Y'
\end{bmatrix}
\]

(14.53)

The contents of the \([Z]\) matrix might be best obtained by means of a least squares estimator. This involves recognizing that measurements obtained in the laboratory will be inaccurate and so there are no values of the coefficient in the \([Z]\) matrix which will satisfy all lines of equation (14.54). A residual matrix is developed which defines the ‘errors’ between the left hand and right hand sides of equation (14.54) i.e.

\[
[E]_{z\times 2} = [A]_{z\times 2} - [D]_{z\times n}[Z]_{n\times 2}
\]

(14.55)

The contents of the \([Z]\) matrix are defined as being those values, which ensure that the sums of the squares of the elements in the \([E]\) matrix are minimized. On multiplying equation (14.55) by \([D]_n^{T}\), we get

\[
[D]^{T}[D][Z] = [D]^{T}[A] \quad \text{or} \quad [Z] = ([D]^{T}[D])^{-1}[D]^{T}[A]
\]

(14.56)

Since measured terms used to make up the \([D]\) and \([A]\) matrices are obtained via the Fourier transformation of the output and input signals. Hence, the noise occurring at a frequency greater than \( n\omega_0 \) is automatically filtered out of the analysis.

With the advent of sophisticated modal analysis equipment, sensors and signal processing hardware and software exciting the system and measuring the corresponding response is convenient, satisfactory and reliable. In laboratory test rigs (both small and full-scale) exciting the system and
measuring the response is fairly easy and convenient. One should be able to excite all possible modes of the system and dynamic characterisation of system could be done fairly satisfactorily. But the application of these excitation methods in real machinery and in situ has not given satisfactory results as yet.

### 14.8 Use of Centrifugal Forces

One of the simplest ways of exciting a journal in a sinusoidal manner is by means of centrifugal forcing, simply by attaching unbalance masses of known magnitude to a rotating shaft. Advantage with this method is that there is no need for costly electromagnetic exciter and the rotational speed dependency of the bearing dynamic characteristics can be identified relatively easily. In present method the processing of measured data is performed in time domain. Out of three methods in two methods an unbalance of known magnitude attached to the journal (and are based on the assumption that inherent rotor unbalance is insignificantly small). The third method involves use of a separate unbalance mass shaft, which can rotate at frequencies independent of the journal rotational frequency.

#### (i) Unbalance mass attached to the journal

Hagg and Sankey (1956, 1958) were among the first to experimentally measure the oil-film elasticity and damping for the case of a full journal bearing with unbalance force only. They used the experimental measurement technique of Stone and Underwood (1947) in which the vibration amplitude and phase of the journal motion relative to the bearing housing were measured using the vibration diagram. They considered only the direct stiffness and damping coefficients along the principal directions (i.e. major and minor axes of the journal elliptical orbit). The equation of motion of a journal having mass, \( m \), can be written as

\[
\begin{align*}
    m\ddot{\xi} + c_{\xi\xi}\dot{\xi} + k_{\xi\xi}\xi &= F \cos(\omega t + \phi) \\
    m\ddot{\eta} + c_{\eta\eta}\dot{\eta} + k_{\eta\eta}\eta &= F \sin(\omega t + \phi)
\end{align*}
\]

where the \( \xi, \eta \) co-ordinates correspond to the major and minor axes of the journal elliptical orbit, \( F \) is a rotating unbalance force amplitude and \( \phi \) is the phase between the journal inertia force and the unbalance force. From the steady-state solution of equation (14.57) and applying the conditions i.e. at \( \eta = 0, \xi = \xi_0, \phi = \phi_0 \) and at \( \xi = 0, \eta = \eta_0, \phi = \phi_0 \) the following simple expressions can be deduced...
(14.58) The stiffness and damping coefficients may be obtained on the basis of the measured unbalance response whirl orbit. Since the cross-coupled coefficients are ignored, the results therefore represent some form of effective rotor-bearing coefficients and not the true film coefficients.

(ii) Alternatively, this means of exciting the journal can be used to determine the bearing oil-film damping coefficients when the bearing stiffness coefficients are already known. (For example by the static force method). The experimental involves measurement of the horizontal and vertical displacement amplitudes of the journal relative to the bearing, and of the bearing or pedestals itself relative to space (fixed foundation). In addition, measurements of the corresponding phase lag angles of each of these displacements behind the unbalance force vector are also made.

![Figure 14.11 A rotor-bearing system with an unbalance](image)

In addition to unbalance force as shown in Figure 14.11, the rotor also has fluid-film forces acting on it, these being transmitted to rotor by shaft. Governing equations of the rotor can be written as

\[ 2f_x - 2k_{xx}x - 2k_{xy}y - 2c_{xx} \dot{x} - 2c_{xy} \dot{y} = 2m(\ddot{x} + \ddot{x}_p) \]

and

\[ 2f_y - 2k_{yx}x - 2k_{yy}y - 2c_{yx} \dot{x} - 2c_{yy} \dot{y} = 2m(\ddot{y} + \ddot{y}_p) \]  

(14.59)
where \( x \) and \( y \) are displacements of journal relative to bearing (or pedestal), \( x_p \) and \( y_p \) are the displacement of pedestal (or bearing) relative to fixed foundation, \( 2m \) is the rotor mass (symmetric) and \( f(t) \) is the known unbalance force on the rotor. We can write

\[
\begin{align*}
    f_x &= F_x e^{i\omega t}; \\
    f_y &= F_y e^{i\omega t} \tag{14.60}
\end{align*}
\]

so that

\[
\begin{align*}
    x &= Xe^{i\omega t}; \\
    y &= Ye^{i\omega t}; \\
    x_p &= X_p e^{i\omega t}; \\
    y_p &= Y_p e^{i\omega t} \tag{14.61}
\end{align*}
\]

where \( F_x, F_y, X, Y, X_p \) and \( Y_p \) are in general complex quantity and contain the amplitude and phase information and \( \omega \) is the rotational speed of the rotor. On substituting equation (14.61) into equation (14.59), we get

\[
\begin{align*}
    \left[ Z_{xx} (\omega) \ Z_{xy} (\omega) \right] \begin{bmatrix} X \n Y \end{bmatrix} &= \begin{bmatrix} F_x + m\omega^2 (X + X_p) \n F_y + m\omega^2 (Y + Y_p) \end{bmatrix} \tag{14.62}
\end{align*}
\]

with

\[
Z = k + j\omega c
\]

On separating real and imaginary terms, we get

\[
\begin{align*}
    \begin{bmatrix}
    m\omega^2 - k_{xx} & k_{xy} & \omega c_{xx} & \omega c_{xy} \\
    -k_{xy} & k_{yy} - m\omega^2 & \omega c_{yy} & \omega c_{yx} \\
    \omega c_{xx} & -\omega c_{xy} & k_{xx} - m\omega^2 & k_{xy} \\
    \omega c_{yx} & -\omega c_{yy} & k_{yx} & k_{yy} - m\omega^2 \\
    \end{bmatrix}
    & \begin{bmatrix} X \n Y \end{bmatrix} = \begin{bmatrix} -m\omega^2 X_p \n -m\omega^2 Y_p \n F + m\omega^2 Y_p \n F + m\omega^2 X_p \end{bmatrix} \tag{14.63}
\end{align*}
\]

Quantities \( m, F_x, F_y, \omega, X, Y, X_p \) and \( Y_p \) are either known or are measured during the course of the experiment. \( k_{xx}, c_{xx}, \cdots \) are unknown (eight for the present case). Equation (14.61) has four equations so if four stiffness coefficients are known (by static force method) the remaining four damping coefficient can be obtained from this. Alternatively, if bearing dynamic parameters are speed-independent then measurement at least two speed will be sufficient to obtain all eight coefficients or by changing \( F \) and rotating the rotor at same speed (for speed-dependent bearing parameters) all eight coefficients can be obtained.
**Ill-Conditioning:** Lee and Hong (1989) identified the bearing dynamic coefficients by using unbalance response measurements of rigid rotor systems supported by two anisotropic bearings. They identified bearing dynamic coefficients, consisting of four damping and four stiffness coefficients, utilising unbalance response measurements from four sensors at two locations for two different trial unbalance conditions. They expressed the unbalance response as two synchronous vibrations, forward and backward; that is

\[ q(t) = Q_f e^{i\omega t} + Q_b e^{-i\omega t} \]  

(14.64)

where \( Q_f \) and \( Q_b \) are the forward and backward whirl response vectors, respectively. Then, from the expression of the form of equation (14.62) the unbalance response can be written as

\[
\begin{bmatrix}
Z_{ff} & Z_{fb} \\
Z_{bf} & Z_{bb}
\end{bmatrix}
\begin{bmatrix}
Q_f \\
Q_b
\end{bmatrix} =
\begin{bmatrix}
F_c \\
0
\end{bmatrix}
\]  

(14.65)

where \( Z \) is the dynamic stiffness matrix, \( F_c \) is the unbalance force vector, and subscripts \( f \) and \( b \) refer to the forward and backward whirl, respectively. Equation (14.64) can be written as

\[
\begin{bmatrix}
Q_f \\
Q_b
\end{bmatrix} =
\begin{bmatrix}
R_{ff} & R_{fb} \\
R_{bf} & R_{bb}
\end{bmatrix}
\begin{bmatrix}
F_c \\
0
\end{bmatrix} =
\begin{bmatrix}
R_{ff} \\
R_{bf}
\end{bmatrix}
F_c
\]  

(14.66)

When the backward whirl response due to unbalance is no longer observed, than \( Q_b = 0 \) and leads to, noting the second set of equation (14.65), \( Z_{bb} = 0 \) and this is the condition of bearing isotropy. Conversely, the information regarding the backward whirl response is lost when bearing exhibits the isotropic condition. That is why the isotropic bearings suffer from the lack of information necessary to identify uniquely all the bearing dynamic parameters (as it was evident in works of Stanway, 1983; and Sahinkaya and Burrows, 1984b). Tiwari et al. 2001 tackled the problem of isotropic bearings by using two methods. Firstly, by using regularisation (Tikhonov and Arsenin, 1977) with the condition of isotropic bearings and secondly, by using the unbalance responses corresponding to rotation of the shaft in both directions (i.e. clockwise and anticlockwise). The second method ensures the information from the forward and backward whirl in the responses. For fluid-film bearings where the dynamic characteristics change for the reverse direction of rotation of the rotor an external auxiliary means of excitation described by Muszynska and Bently (1990) can be used.

**(iii) Unbalanced mass attached to an independent vibrator shaft**

In previous method, equation (14.61) has eight unknowns \( (k_{xx}, c_{xx}, \cdots) \) and it has four equations. If a test rig capable of providing excitation by means of unbalance forcing, where the forcing frequency could be varied without upsetting the journal rotational frequency. Thus a second equation (14.61)
could be obtained, resulting in eight simultaneous equations in all, by using a different value of $\omega$ without upsetting the bearing Sommerfeld number. Two sets of measurements can be taken for two different rotational frequency of secondary shaft, most convenient would be $\pm \omega$ so that the steady state position should not disturb (as shown in Figure 14.12b).

![Diagram](image)

Figure 14.12 An anti-synchronous excitation by an auxiliary unbalance unit

**Example 14.5** For estimation of bearing dynamic coefficients the following measurements were made: (i) $X_1$ and $Y_1$ for simultaneous application of $F_{x_1}$ and $F_{y_1}$ and (ii) $X_2$ and $Y_2$ for simultaneous application $F_{x_2}$ and $F_{y_2}$; where $X$ and $Y$ are displacements and $F$ is force and in general they all are complex in nature. If shapes of both the orbits of the shaft center are circular in shape, whether it would be possible to estimate all bearing dynamic coefficients from these two measurements.

**Solution:** Consider a single bearing and use a complex stiffness, $Z = k - m\omega^2 + j\omega\epsilon$, at a single frequency to describe the equation of motion in frequency domain, as
Using two unbalance runs with corresponding responses \( X_1, Y_1, X_2 \) and \( Y_2 \) with right hand sides \( F_{x1}, F_{y1}, F_{x2} \) and \( F_{y2} \), equation (a) may be written as

\[
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
Y_1
\end{bmatrix} = \begin{bmatrix}
F_{x1} \\
F_{y1}
\end{bmatrix}
\]

(b) The solution of equation (b) is obtained as

\[
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
= \frac{1}{(X_1Y_2 - X_2Y_1)} \begin{bmatrix}
F_{x1} & F_{x2} \\
F_{y1} & F_{y2}
\end{bmatrix}
\begin{bmatrix}
Y_2 - X_2 \\
Y_1 - X_1
\end{bmatrix}
\]

(c) For circular orbits \( Y_1 = jX_1 \) and \( Y_2 = jX_2 \) (or both negative, depends on the definition of axes, and the direction of rotation). Then the denominator of equation (c) becomes

\[
X_1Y_2 - X_2Y_1 = X_1(jX_2) - X_2(jX_1) = 0
\]

(d) and hence, equation (d) is ill-conditioned for circular orbits. Having a third unbalance run does not help. For three unbalances equation (b) may be written as

\[
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
Y_1
\end{bmatrix} = \begin{bmatrix}
F_{x1} & F_{x2} & F_{x3} \\
F_{y1} & F_{y2} & F_{y3}
\end{bmatrix}
\]

(e) The least squares solution involves the following inversion

\[
\begin{bmatrix}
X_1 & X_2 & X_3 \\
Y_1 & Y_2 & Y_3
\end{bmatrix}
= \begin{bmatrix}
X_1^2 + X_2^2 + X_3^2 & X_1Y_1 + X_2Y_2 + X_3Y_3 \\
X_1Y_1 + X_2Y_2 + X_3Y_3 & Y_1^2 + Y_2^2 + Y_3^2
\end{bmatrix}^{-1}
\]

\[
= \frac{1}{(Y_1^2 + Y_2^2 + Y_3^2)(Y_1^2 + Y_2^2 + Y_3^2) - (X_1Y_1 + X_2Y_2 + X_3Y_3)} \begin{bmatrix}
Y_1^2 + Y_2^2 + Y_3^2 & -(X_1Y_1 + X_2Y_2 + X_3Y_3) \\
X_1Y_1 + X_2Y_2 + X_3Y_3 & X_1^2 + X_2^2 + X_3^2
\end{bmatrix}
\]

(f)
If \( Y_i = j X_i \), then the denominator of the equation (f) becomes

\[
(X_1^2 + X_2^2 + X_3^2)(Y_1^2 + Y_2^2 + Y_3^2) - (X_1 Y_1 + X_2 Y_2 + X_3 Y_3)^2
\]

and the circular orbits are ill-conditioned. There is another possibility when ill-conditioning may occur, namely when \( Y_i = \alpha X_i \) and \( Y_2 = \alpha X_2 \) for any value of \( \alpha \), where \( \alpha \) is a constant. Then the denominator of equation (c) becomes zero, leading to ill-conditioning. This means that a change in orbit from one unbalance to the next is required. The ill-conditioning due to a circular orbit may be avoided by taking measurements in both the clockwise and anticlockwise directions of rotation of the rotor. For this case \( Y_1 = j X_1 \) and \( Y_2 = -j X_2 \). Then the denominator of equation (c) becomes

\[
X_1 Y_2 - X_2 Y_1 = X_1 (-j X_2) - X_2 (j X_1) \neq 0
\]

and hence, equation (c) becomes well-conditioned.

Since in practice the run-down/up synchronous responses from large machinery are readily available, the application of the methods described in this section to the run-down/up synchronous responses of such machines is the great need for the future.

**Example 14.6** Determine the dynamic coefficients (i.e. the stiffness and damping coefficients) of a bearing for the following independent measurements, which was taken at a constant speed of 2000 rpm: (i) for an unbalance mass of 2 gm at 20° it gives displacements in the vertical and horizontal directions, respectively, of 20 μm with phase of 120° and 16 μm with phase of 330°, (ii) for another unbalance mass of 4 gm at 190° it gives displacements in the vertical and horizontal directions, respectively, of 25 μm with phase of 60° and 20 μm with phase of 230°. All the unbalance masses are at 3 cm of radius.

**Solution:** From equation (c) of example 14.5, we have

\[
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix} = \frac{1}{(X_1 Y_2 - X_2 Y_1)} \begin{bmatrix}
F_{x1} & F_{y1} \\
F_{x2} & F_{y2}
\end{bmatrix} \begin{bmatrix}
Y_2 & -X_2 \\
Y_1 & X_1
\end{bmatrix}
\]
All the measurements are at $\omega = 33.33$ rad/s. From the first measurement, we have

$$F_{x1} = ma \sqrt{ee^{i\theta}} = 2 \times 10^{-3} \times 0.03 \times e^{i(20\pi/180)} = 6.0 \times 10^{-5} e^{i0.3491} \text{ N}$$

$$Y_1 = 16 \times 10^{-6} e^{i(330\pi/180)} = 16 \times 10^{-6} e^{i5.7596} \text{ m}$$

$$X_1 = 20 \times 10^{-6} e^{i(120\pi/180)} = 20 \times 10^{-6} e^{i2.0944} \text{ m}$$

From the second measurement, we have

$$F_{x2} = 4 \times 10^{-3} \times 0.03 \times e^{i(190\pi/180)} = 12 \times 10^{-5} e^{i3.161} \text{ N}$$

$$Y_2 = 25 \times 10^{-6} e^{i(60\pi/180)} = 25 \times 10^{-6} e^{i1.0472} \text{ m}$$

$$X_2 = 20 \times 10^{-6} e^{i(230\pi/180)} = 20 \times 10^{-6} e^{i4.0143} \text{ m}$$

Hence, we have

$$X_1 Y_2 - X_2 Y_1 = \left(20 \times 10^{-6} e^{i2.0944}\right) \times \left(25 \times 10^{-6} e^{i1.0472}\right) + \left(20 \times 10^{-6} e^{i4.0143}\right) \times \left(16 \times 10^{-6} e^{i5.7596}\right)$$

$$= 5.0 \times 10^{-10} e^{i3.1416} + 3.2 \times 10^{-10} e^{i9.7739} = -1.9930 \times 10^{-10} + j1.0946 \times 10^{-10} \text{ m}^2$$

On substituting in equation (a), we get

$$\begin{bmatrix} Z_{x_1} & Z_{y_1} \\ Z_{x_2} & Z_{y_2} \end{bmatrix} = \frac{1}{(X_1 Y_2 - X_2 Y_1)} \begin{bmatrix} F_{x1} & F_{x2} \\ F_{y1} & F_{y2} \end{bmatrix} \begin{bmatrix} Y_2 & -X_2 \\ Y_1 & X_1 \end{bmatrix}$$

$$= \begin{bmatrix} 10.4685 - j4.9574 & -9.0332 - j1.3943 \\ -4.9574 - j10.4685 & -1.3943 + j9.0332 \end{bmatrix}$$

Hence, we have

$$k_{x_1} = \text{Re}(Z_{x_1}) = 10.4685 \text{ N/m}, \quad k_{y_1} = \text{Re}(Z_{y_1}) = -1.3943 \text{ N/m}$$
\[ k_{yx} = \text{Re}\left(Z_{xy}\right) = -9.0332 \text{ N/m} \quad k_{yx} = \text{Re}\left(Z_{yx}\right) = -4.9574 \text{ N/m} \]

\[ c_{yx} = \frac{1}{\omega} \text{Im}\left(Z_{xy}\right) = -0.1487 \text{ N-s/m} \quad c_{yx} = \frac{1}{\omega} \text{Im}\left(Z_{yx}\right) = 0.2710 \text{ N-s/m} \]

\[ c_{yy} = \frac{1}{\omega} \text{Im}\left(Z_{yy}\right) = -0.0418 \text{ N-s/m} \quad c_{yx} = \frac{1}{\omega} \text{Im}\left(Z_{yx}\right) = -0.3141 \text{ N-s/m} \]

In modern power plants because of ever-increasing demand for high power and high speed with uninterrupted and reliable operation, the accurate prediction of the dynamic behaviour of such machinery has become increasingly important. The most crucial part of such large turbo-generators is the machine elements that allow relative motion between the rotating and the stationary machine elements i.e., the bearings. Historically the theoretical estimates of the dynamic bearing characteristics have always been a source of error in the prediction of dynamic behaviour of rotor-bearing systems. Consequently, accurate parameter identification is required to reduce the discrepancy between the measurements and the predictions. In particular, physically meaningful experimental identification of bearing dynamic coefficients is necessary because of the difficulty in accurate system modelling and analysis (Smart, 1998).

Obtaining reliable estimates of the bearing static load in actual test conditions is quite difficult and this leads to inaccuracies in the well-established theoretical bearing models. Hence the estimation of bearing dynamic parameters in actual test conditions is important to rotor design. Mitchell et al. (1966) obtained the stiffness of an oil film bearing experimentally, by application of static loads. Morton (1971) devised the measurement procedure for estimation of the dynamic characteristics of a large sleeve bearing by application of dynamic loads (sinusoidal excitation at a frequency non-synchronous with the running frequency of the sleeve) using vibrators, whilst Childs and Hale (1993) devised a test apparatus and facility to identify the rotodynamic coefficients of high speed hydrostatic bearings (with shakers to provide sinusoidal excitation along two perpendicular directions). Nordmann and Schollhorn (1980) identified the stiffness and damping coefficients of journal bearings whereas Kraus et al. (1987) identified the coefficients for rolling element bearings by means of the impact method. Sahinkaya and Burrows (1984) and Tieu and Qiu (1994) estimated the linearised oil film parameters from the out-of-balance response where the shaft was excited by a known unbalance force (synchronous excitation). Chen and Lee (1997) identified rolling element dynamic characteristics in flexible rotor-bearing systems by using unbalance responses at all bearings and several shaft locations without a priori knowledge of the unbalance. Muszynska and Bently (1990)
developed a *perturbation technique* (two frequency swept periodic inputs) for estimation of these parameters. Tiwari and Vyas (1995) extracted the non-linear stiffness parameters of rolling element bearings based on the *natural random response* at the bearings of rotor-bearing systems. Goodwin (1991) reviewed the experimental approaches to rotor support impedance measurement. Swanson and Kirk (1997) presented a survey in tabular form of the experimental data available in the open literature for fixed geometry hydrodynamic journal bearings.

Most of the bearing parameter identification methods available require the bearing to be tested in isolation or in a rotor-bearing system where the shaft is rigid. Very few researchers have considered the flexibility of the shaft.

**14.9 Transient Methods**

Until the early 1970s the usual method for testing dynamic characteristics was sinusoidal excitation. In 1971 Downham and Woods proposed a technique using a pendulum hammer to apply an impulsive force to a machine structure. Although they were interested in vibration monitoring rather than the determination of bearing dynamic coefficients, their work was of interest because impulse testing was thought to be capable of exciting all the modes of a linear system. Due to the wide application of the FFT algorithm and the introduction of the hardware and software signal processor, the testing of dynamic characteristics by means of transient excitation is now common. In this method, it is possible to take measurement on running machines. In the present case, let the system consists of a symmetrical rigid rotor mounted in two identical journal bearings. Transient vibration of the rotor in the bearings is caused by applying a force impulse (an impulse is as shown in Figure 14.13 or a step forcing as shown in Fig. 14.14) to the rotor centre of gravity. In practice this is provided by striking the rotor with a ‘calibrated’ hammer whose head mass is known. This means of excitation results in an impulse, which lasts for a finite period of time (typically a fraction of second). If an accelerometer is mounted in the hammerhead, it is possible to determine the instantaneous force, which is applied to the rotor. The electrical output from the hammer will then indicate the vibration of the applied force with time. An impulse can be considered as made up of a number of sine waves of different frequencies, all occurring simultaneously. By varying the hammer head mass, the stiffness of the hammer impact force (tip) and the initial hammer head velocity, it is possible to vary the amplitude, frequency content and duration of the applied impulse. Equations of motion the journal would be

\[
\begin{align*}
    f_x - k_{xx} x - k_{xy} y - c_{xx} \dot{x} - c_{xy} \dot{y} &= m \ddot{x} \\
    f_y - k_{yx} x - k_{yy} y - c_{yx} \dot{x} - c_{yy} \dot{y} &= m \ddot{y}
\end{align*}
\]  

(14.67)
Since forcing may be considered to be sinusoidal, albeit at several different frequencies, any one component will be of the form in the horizontal and vertical directions

\[ f_x = F_x e^{j\omega t} \quad \text{and} \quad f_y = F_y e^{j\omega t} \]  \hspace{1cm} (14.68)

The corresponding horizontal and vertical displacements \( x \) and \( y \) will be of the form

\[ x = X e^{j\omega t} \quad \text{and} \quad y = Y e^{j\omega t} \]  \hspace{1cm} (14.69)

so that

\[ x = j\omega x; \quad \ddot{x} = -\omega^2 x; \quad \dot{y} = j\omega y \quad \text{and} \quad \ddot{y} = -\omega^2 y \]  \hspace{1cm} (14.70)

On substituting in equation of motion (14.67) yields

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} = \begin{bmatrix}
(k_{xx} - m\omega^2 + j\omega c_{xx}) & (k_{xy} + j\omega c_{xy}) \\
(k_{yx} + j\omega c_{yx}) & (k_{yy} - m\omega^2 + j\omega c_{yy})
\end{bmatrix} \begin{bmatrix}
X \\
Y
\end{bmatrix} \hspace{1cm} (14.71)
\]

which can written as
\[
\begin{pmatrix}
X \\
Y
\end{pmatrix} = \frac{1}{D} \begin{pmatrix}
(k_{yy} - m\omega^2 + j\omega\epsilon_{yy}) & -(k_{yx} + j\omega\epsilon_{yx}) \\
-(k_{xy} - j\omega\epsilon_{xy}) & (k_{xx} - m\omega^2 + j\omega\epsilon_{xx})
\end{pmatrix} \begin{pmatrix}
F_x \\
F_y
\end{pmatrix}
\]

with
\[
D = (k_{xx} - m\omega^2 + j\omega\epsilon_{xx})(k_{yy} - m\omega^2 + j\omega\epsilon_{yy}) - (k_{yx} + j\omega\epsilon_{yx})(k_{xy} + j\omega\epsilon_{xy})
\]

Equation (14.70) is similar to the case of electromagnetic exciter method (first method) except in present case the inertia force has now also been allowed for. If forcing is applied in one direction (for example the hammer strikes the rotor in horizontal direction) then it is possible to define the reacceptance as: (from equation (14.70))

\[
R_{sx} = \frac{(k_{yy} - m\omega^2 + j\omega\epsilon_{yy})}{D} = \frac{X}{F_x} \quad \text{and} \quad R_{sy} = \frac{-(k_{yx} + j\omega\epsilon_{yx})}{D} = \frac{Y}{F_x}
\]

Similarly if hammer strikes the rotor in the y-direction then

\[
R_{sy} = \frac{-(k_{yx} + j\omega\epsilon_{yx})}{D} = \frac{X}{F_y} \quad \text{and} \quad R_{sx} = \frac{(k_{xx} - m\omega^2 + j\omega\epsilon_{xx})}{D} = \frac{Y}{F_y}
\]

The reacceptance terms defined in equations (14.73) and (14.74) are clearly functions of frequency and so take a different value depending on the vibration frequency being considered. The reacceptance terms are in general complex because displacement and force are not in-phase. The method of determining the fluid-film stiffness and damping coefficient makes use only of the modulus of the reacceptance terms, however, does not use the data describing phase. In experiment the right hand side of equations (14.73)-(14.74) exist at many different frequencies simultaneously, and the corresponding receptance terms must be determined for each of these frequencies. The reacceptance will be

\[
R_{sx}(\omega) = \frac{\text{Fourier transform of } x}{\text{Fourier transform of } f_x} = \frac{\int x(t)e^{-j\omega t} dt}{\int f_x(t)e^{-j\omega t} dt}
\]

This may be obtained in experiment by spectrum analyzer and a typical reacceptance is shown in Figure 14.15.
The above reacceptances have been obtained from right hand side of equations (14.73)-(14.74) using two independent forcing. Now our aim is to obtain $k_{xx}$ etc. so that when it is substituted back in left hand side of equation (14.73)-(14.74) it should give the value of the right hand side of equation (14.73)-(14.74). These processes can be repeated until appropriate values are found which results in the difference between left hand side and right hand side of equations (14.73)-(14.74) being minimized, for all frequencies under consideration. The “least squares error” criteria may be used so that to minimize a scalar quantity

$$s = \sum_i \sum_j \sum_\omega [R_{ij}(\omega)_{\text{theory}} - R_{ij}(\omega)_{\text{exp}}]^2$$ \hspace{1cm} (14.76)

**Practical Issues:** Using a filter the synchronous unbalance response must be subtracted as shown in Figure 14.16 from the transient responses.

![Figure 14.16 Effect of residual unbalance on the impulse response](image)

The step function can be generated by giving gradual static load to the rotor and suddenly releasing the load at well defined upper limit of the static load as shown in Figure 14.14. It is especially suitable for heavy rotors in industries to give transient excitations.

**Flexible Rotors:** Analytically obtained bearing dynamic characteristics have been a major source of error in the modelling and analysis of large turbo-generators. It is for this reason that designers of high-speed rotating machinery mostly rely on experimentally derived values for support stiffness and
damping in their calculations. The present work is aimed at developing an identification method for bearing dynamic coefficients by using the rotor-bearing system’s responses for a known (measurable) impact force.

Several time domain and frequency domain techniques have been developed for determining the fluid-film bearing dynamic coefficients. Many works dealt with identification of bearing coefficients and rotor-bearing system coefficients using impulse, step change in force, and synchronous and non-synchronous excitation techniques. Among all the experimental methods, the impact excitation method proposed by Nordmann and Scholhorn (1980) to identify the stiffness and damping coefficients of journal bearings is the most economical and convenient. Chan and White (1980) used the impact method to identify bearing dynamic coefficients in a rotor mounted on two symmetric bearings by curve-fitting frequency responses. Since in many rotor-bearing systems bearings are not symmetric, the assumption of the symmetry limits the application of the method. Arumugam et al. (1995) extended the method of structural joint parameter identification proposed by Wang and Liou (1991) to identify the linearised oil-film coefficients utilizing the experimental frequency response functions (FRFs) and theoretical FRFs obtained by finite element modelling. This method was used to identify the linearised oil-film coefficients of tilting pad cylindrical journal bearings. Qiu and Tieu (1997) extended the impact excitation method proposed by Nordmann and Scholhorn (1980) to estimate dynamic coefficients of two asymmetric bearings from impulse response of a rigid rotor bearing system. Flexibility of the rotor was not considered in this method.

Goodwin (1991) reviewed the experimental approaches to rotor support impedance measurement. He concluded that measurements made by multi-frequency test signals provide more reliable data. Swanson and Kirk (1997) presented a survey in tabular form of the experimental data available in the open literature for fixed geometry hydrodynamic journal bearings. Recently, Tiwari et al. (2004) wrote an extensive review of the experimental identification of the dynamic coefficients of bearings in a rotating machine. Major emphasis was given to vibration based identification methods and the review encompassed descriptions of experimental measurement techniques, mathematical modelling, parameter extraction algorithms and uncertainty in the estimates applied to a variety of bearings. A chronological list of source material on the estimation of the dynamic coefficients of bearings from experimental data with brief details was also given in tabular form. The present paper is an extension of the author’s work (2002), which dealt with the estimation of bearing coefficients based on rundown data.

Since impact tests have to be conducted for each rotor speed at which bearing dynamic parameters are desired, this method is time consuming. In general, the governing equations for a bearing include
coupling between the two perpendicular directions and this limits the amount of information that can be extracted from a single impulse test (Burrows and Stanway, 1977). Errors in the estimates will be more for the case when bearing dynamic coefficients are function of external excitation frequency apart from function of rotor rotational frequency. Impulse testing may lead to underestimation of input forces when applied to a rotating shaft as a result of the generation of friction-related tangential force components (Tonnesen and Lund, 1988; Muszynska et al., 1993) and, further, is prone to poor signal-to-noise ratios because of the high crest factor.

14.10 Methods using Unknown Excitation

In industrial machinery the application of a calibrated force is difficult to apply. The presence of inherent forces in the system, due to residue unbalance, misalignment, rubbing between rotor and stator, aerodynamic forces, oil-whirl, oil-whoop and other instability, make it worse to have assessment of the forcing given to the system. In such cases the accurate information available is of the response only. Adams and Rashidi (1985) used bearing stiffness coefficients measured using the static loading method and measured orbital motion at an adjustable threshold speed to extract bearing damping coefficients by inverting the associated eigen problem. The approach stems from the physical requirement for an exact internal energy balance between positive and negative damping influences at an instability threshold. This approach does not require the measurement of dynamic forces. The approach was illustrated by simulation.

Chen and Lee (1997) identified the eight-linearised dynamic characteristics of rolling element bearings using finite element modelling of rotor-bearing systems. From a finite element model, equation (14.24), they developed an algorithm for the bearing dynamic coefficient estimation, similar to that of the regression equation (14.33). The algorithm needs the measurement of the shaft displacements at all nodes (including bearings and discs) defined in the finite element model. The bearing dynamic coefficients were estimated in time domain. The measurements at shaft nodes eliminated the knowledge of the forcing a priori, since their main intention was to develop an estimation method that could be used on-line. They used two sets of measurements corresponding to two spin speeds near the rotor speed at which the parameters were being estimated. The developed estimator had an inherent bias problem. The measured signals were reconstructed by signal processing to obtain unbiased estimates of the response which were used to get unbiased estimated of the bearing coefficients. They compared their results indirectly by measuring FRFs by using an independent impact test and simulating the FRFs using the experimentally obtained bearing parameters and known rotor model. This method had the practical difficulty of measuring the rotor response at all node locations apart from the time consuming signal processing. Chen and Lee (1995) extended the bearing

* Advanced topic
dynamic parameter estimation method in the frequency domain for three different cases when (a) the rotor model was completely known (b) the rotor element in the vicinity of the bearing was completely known and (c) the rotor element mass in the vicinity of the bearing was known. But their algorithm needs the measurement of displacements and angular displacements at all node points for cases (b) and (c), which puts practical limitations on the method especially because the accurate measurement of angular displacement is very difficult. Lee and Shih (1996) presented an estimation procedure for finding rotor parameters including bearing dynamic coefficients, shaft unbalance distributions and disc eccentricity in flexible rotors based on the transfer matrix method (Lee et al. 1991 and 1993). The normal equations were formulated by using the relations between measured response data and the known system parameters; then the parameter estimation were performed using the least squares method by assuming that the bearing dynamic coefficients to be constant at close spin speeds. The procedure was demonstrated by numerical simulations.

Tiwari and Vyas (1995, 1996) offered a means of estimating the bearing non-linear stiffness without explicit force measurements, based on the analysis of the random response signals measured at the bearing caps, provided that the system may be assumed to be perfectly balanced. The rotor-bearing system was modeled as a SDOF model through the Fokker-Planck equation and the vibrations resulting from random imperfections of the bearing surfaces and assembly, were processed through a curve-fitting algorithm to obtain the necessary bearing stiffness parameters. The method does not require an estimate of the excitation forces and works directly on the measured response signals of the system. The algorithm was illustrated on a laboratory rotor-bearing test rig and the results were compared with those obtained through an existing analytical model. Subsequently the method was extended to the flexible rotor-bearing multi-DOF systems (Tiwari and Vyas, 1997a and 1998) and for the case of combined random and sinusoidal (residual unbalance) excitations (Tiwari and Vyas, 1997b). Based on the similar approach, Tiwari (2000) extended the method to the identification of the form of the non-linearity (i.e. softening or hardening type) in the restoring (stiffness) force as well as its parameter estimation for the rolling element bearing. The following simple estimation expression was derived:

\[
k(x) = -\frac{\sigma_i^2}{p(x)} \frac{dp(x)}{dx}
\]

(14.77)

The above equation is a representation of the restoring force function, \( k(x) \), in terms of the displacement and velocity response of the rotor-bearing system. Velocity variance, \( \sigma_v^2 \), probability function, \( p(x) \), and its derivative, \( dp(x)/dx \), can be computed from the experimentally measured
displacement and velocity data ($x$ and $\dot{x}$) in time domain, which enables the reconstruction of the unknown function $k(x)$.

Odiari and Ewins (1992) used Volterra and Wiener based techniques for the identification of non-linear dynamic parameters of rotor-bearing systems. It was shown by simulation studies that the frequency based approach produces more accurate results than their time-domain equivalents. Also the computation time is less for frequency domain methods. Khan and Vyas (1999) extended the non-parametric Volterra kernel identification procedure, to non-linear bearing stiffness estimation of a single DOF rotor-bearing system. Subsequently in 2001 (a and b) the identification procedure was extended to a more general multi-DOF rotor-bearing system. The procedure was illustrated using the numerical simulation.

Attempts have been made to identify foundation parameters along with unbalance in flexible rotor-bearing-foundation systems with known rotor and bearing models (Lees, 1988; Zanetta, 1992; Lees and Friswell, 1997; Smart, 1998; Edwards et al., 2000; Sinha et al., 2001). There is a need to extend this method to unknown bearing models.

The problem of identification of the bearing dynamic parameters along with the unbalance information, when only response information is available has great potential. These can be used for an on-line prediction of the system dynamic behaviour for the condition monitoring purpose. Seals have some specialised methods for rotor dynamic parameter estimation and the next section touch upon these methods briefly.

**14.11 Special Methods of Estimation of Dynamic Parameter of Seals**

A model of a typical annual (or clearance) seal is shown in Fig. 14.17(a). The geometrical shape of a clearance seal is similar to that of a hydrodynamic bearing; however, they are different in the following aspects. To avoid contact between a rotor and a stator, the ratio of the clearance to the shaft radius in seals is made few times (2 to 10 times) larger than that of hydrodynamic bearings. The flow in seals is turbulent and in hydrodynamic bearings it is laminar. Therefore, unlike hydrodynamic bearing, one cannot use the Reynolds equation for analysis of seals (refer Chapter 3 for more details). When a rotor vibrates inside the seal then a reaction force of the fluid-film acts on it. In case of a small vibration around the equilibrium position, the fluid force can be linearized on the assumption that deflections $\Delta x$ and $\Delta y$ are small. The general governing equations of fluid-film forces on seals,

---

* Advanced topic
which has small oscillations relative to the rotor, is given by the following linearized force-displacement model (Childs et al., 1986)

\[
-\begin{bmatrix}
 f_x \\
 f_y \\
\end{bmatrix} = \begin{bmatrix}
 k_{xx} & k_{xy} \\
 k_{yx} & k_{yy} \\
\end{bmatrix} \begin{bmatrix}
 \Delta x \\
 \Delta y \\
\end{bmatrix} + \begin{bmatrix}
 c_{xx} & c_{xy} \\
 c_{yx} & c_{yy} \\
\end{bmatrix} \begin{bmatrix}
 \Delta \dot{x} \\
 \Delta \dot{y} \\
\end{bmatrix} + \begin{bmatrix}
 m_{xx} & 0 \\
 m_{yx} & m_{yy} \\
\end{bmatrix} \begin{bmatrix}
 \Delta \ddot{x} \\
 \Delta \ddot{y} \\
\end{bmatrix}
\]  

(14.78)

where \( f_x \) and \( f_y \) are fluid-film reaction forces on seals in x and y directions. \( k, c, m \) represent the stiffness, damping and added-mass coefficients, subscripts: \( xx \) and \( yy \) represent the direct and \( xy \) and \( yx \) represent the cross-coupled terms, respectively. These coefficients vary depending on the equilibrium position of the rotor (i.e. magnitude of the eccentricity), rotational speed, pressure drop, temperature conditions etc. The off-diagonal coefficients in equation (14.78) arise due to fluid rotation within the seal and unstable vibrations may appear due to these coefficients. Equation (14.78) is applicable to liquid annular seals. But for the gas annular seals, the added mass terms are negligible.

For small motion about a centered position (or with very small eccentricity) the cross-coupled terms are equal and opposite (e.g., \( k_{xy} = -k_{yx} = k_c \) and \( c_{xy} = -c_{yx} = c_c \)) and the diagonal terms are same (e.g., \( k_{xx} = k_{yy} = k_d \) and \( c_{xx} = c_{yy} = c_d \)) (Childs et al., 1986). Considering these relationships and neglecting the cross-coupled added-mass terms, equation (14.78) takes the following form

\[
-\begin{bmatrix}
 f_x \\
 f_y \\
\end{bmatrix} = \begin{bmatrix}
 k_d & k_c \\
 -k_c & k_d \\
\end{bmatrix} \begin{bmatrix}
 \Delta x \\
 \Delta y \\
\end{bmatrix} + \begin{bmatrix}
 c_d & c_c \\
 -c_c & c_d \\
\end{bmatrix} \begin{bmatrix}
 \Delta \dot{x} \\
 \Delta \dot{y} \\
\end{bmatrix} + \begin{bmatrix}
 m_d & 0 \\
 0 & m_d \\
\end{bmatrix} \begin{bmatrix}
 \Delta \ddot{x} \\
 \Delta \ddot{y} \\
\end{bmatrix}
\]  

(14.79)

where subscripts: \( d \) and \( c \) represent direct and cross-coupled, respectively. The rotor dynamic parameters largely affect the performance of the turbomachineries as they lead to serious synchronous and sub-synchronous vibration problems. Whirl frequency ratio, \( f = k_c / (c_d \omega) \) is a useful non-dimensional parameter for comparing the stability properties of seals. For circular synchronous orbits, it provides a ratio between the destabilizing force component due to \( k_c \) and the stabilizing force component due to \( c_d \). In experimental estimation of rotor dynamic parameters of seals, these coefficients (of equations (14.78) and (14.79)) are determined with the help of measured vibrations data from a seal test rig.

### 14.11.1 Experimental Estimation Procedures

In this subsection, various seal test approaches used for the estimation of rotor dynamic parameters for governing equations of the form as equation (14.79) of seals are reviewed with a schematic representation. In the method used by Benkert and Wachter (1980), the seal rotor (Fig. 14.17a) is statically displaced relative to its stator, the circumferential pressure distribution is measured and
integrated, and the resultant reaction force is calculated. This method does not yield any damping values since only static load is applied. Referring to equation (14.79), the static rotor displacement $\Delta x$ in the $x$-direction yields (while keeping $\Delta y = 0$)

$$k_d = \frac{F_x}{\Delta x} \quad \text{and} \quad k_c = \frac{F_x}{\Delta x}$$

(14.80)

---

**Fig. 14.17**

(a) Rotor with static displacement   (b) Eccentric rotor  (c) External shaker excitation

Fig. 14.17b describes the rotor motion relative to the stator that represents two different types of rotor excitation arrangement, i.e. eccentric rotor (Childs and Garcia, 1987) and eccentric sleeves (Kanemori and Iwatsubo, 1992). The varying clearances modulated the local flows in and out of the seal and circumferentially in the seal annulus, and thereby caused the static pressure in the seal annulus to vary circumferentially and periodically. At each instant, the varying component of the static pressure had an essentially sinusoidal distribution around the circumference, and this pressure pattern rotates in synchronism with rotor whirl. The circumferential pressure distribution was measured and integrated to get the resultant reaction forces acting on the rotor. The centered circular orbit is defined by

$$x = e_0 \cos(\Omega t) \quad \text{and} \quad y = e_0 \sin(\Omega t),$$

where $\Omega$ is the frequency of excitation, $e_0$ is the whirl radius and $t$ is the instant time. Hence, equation of the form as equation (14.79) yields the following radial and circumferential coefficient (subscripts: $r$ and $\theta$, respectively) definitions

$$\frac{F_r}{e_0} = -k_d - c_c \omega + m_d \omega^2 \quad \text{and} \quad \frac{F_\theta}{e_0} = k_c - c_d \omega$$

(14.81)

where $k$, $c$ and $m$ represent the stiffness, damping and added-mass coefficients, and subscripts $d$ and $c$ represent the direct and cross-coupled terms, respectively. The test rig is used to measure $F_r/e_0$ and $F_\theta/e_0$ versus Reynolds number and rotor rotational frequency, $\omega$. Iwatsubo and Sheng (1990) and Kaneko et al. (1998) assumed seal rotor dynamic parameters independent of $\omega$ and used the measured $F_r/e_0$ and $F_\theta/e_0$ with respect to $\omega$ for curve-fitting to obtain rotor dynamic parameters, i.e. $k_d$, $k_c$, $c_c$,
$c_d$ and $m_d$. Kanemori and Iwatsubo (1992) used similar procedure with the exceptions that the dynamic moment coefficients were also identified. The form of the moments due to fluid-film was similar to that of equation (14.79) with corresponding moment coefficients (e.g. $k_{\theta\theta}, k_{\theta\phi}$, etc. where $\theta$ is the transverse angular displacement). The rotor was independently driven by two motors to realise the spinning and whirling motion.

For the case when seal rotor dynamic parameters depend upon $\omega$, equation (14.79) can be approximated with the assumption that $k_c$ and $c_c$ vary linearly with $\omega$ as (Childs, 1993)

$$F_r/l_{o} = -k_{df} + m_{df} \omega^2 \quad \text{and} \quad F_{\theta}/e_{o} = -c_{df} \omega$$

(14.82)

When measurements are obtained for fixed axial Reynolds number over a range of $\omega$ the above equations can be used to curve-fit to obtain effective rotor dynamic parameters i.e. $k_{ef}, c_{ef}$ and $m_{ef}$. This approach eliminates rotor dynamic parameters dependency on $\omega$.

Wright (1978, 1983) proposed a method in which a centered circular whirling orbit (Fig. 7c) was obtained by an active feedback system. The conceptual approach (Childs et al., 1986) that is used in estimation of RDPs for small motion about the static eccentricity position, shown in Fig. 7c, defined by the coordinates $(e_o, 0)$. For this position, shaker applies the harmonic horizontal motion with an excitation frequency $\Omega$ (a calibrated imbalance can also be used with $\omega = \Omega$, where $\omega$ is the rotor spin speed or the system (preferably housing) can be excited by an impact of the hammer with a multi-frequency excitation). On substituting measured forces and displacements data into equation (14.78), for a given operating condition, by curve-fitting techniques the rotor dynamic parameters for seals can be obtained. A detailed account of general estimation procedures can be found in Tiwari et al. (2002, 2004).

### 14.11.2 Previous Seal Rotor Dynamic Data and Resources

Allaire and Flack (1982) presented a partial review of the literature on lateral impeller forces by considering the case studies for coolant pumps. Flack and Allaire (1984a) critically reviewed the hydraulically generated lateral forces in pumps. The experimental measurement of pressures, static forces and dynamic forces in pumps were also reviewed. Flack and Allaire (1984b) reviewed the static and dynamic characteristics of tilting pad and turbulent hydrostatic journal bearings. Etison (1982,1985) reviewed the experimental observations and theoretical analyses of face seal dynamics. The more recent textbooks on rotor dynamics include information on rotor dynamic characteristics of rotary seals. Vance (1988), Krämer (1993), Rao (2000) and Adams (2001) provide a good
introductory treatments of seal dynamics. However, one of the first and a comprehensive survey of literatures related to experimental estimation of RDPs of seals was done by Childs (1993). It is the single most, till today, sources of computational and experimental data information and references for seal rotor dynamic characteristics. He reviewed literatures based on different seals geometries with different operating parameters. Swanson and Kirk (1997) carried out an survey of the experimental research on the static and/or dynamic characteristics of fixed geometry, hydrodynamic journal bearings and reported the type(s) of bearing, size of bearing(s) and range of parameters measured in each work. Recently, Tiwari et al. (2004) gave a comprehensive survey on RDPs of bearings, with major emphasis especially on hydrodynamic bearings, with cursory mention about rotary seals.

The present literature survey is aimed at the review of experimental methods for the determination of the RDPs of the seals in rotor-bearing-seal systems, and will hopefully be useful to both the practising engineers and the researchers in this field. For the practicing engineer guidance for simple experimental determination of these parameters with associated uncertainty is offered, whilst researchers may appreciate the diverse methods available and the discussion of their limitations so as to develop improved methods. The review has been presented in accordance with different types of seals geometry viz., plain annular seals, labyrinth seals, helically grooved seals, hole and triangular patterns, honeycomb seals, etc. with some overlapping (hybrid seals) among them.

14.12 Accuracy of Estimated Bearing Coefficients

Uncertainty of the test data is a result of the individual uncertainties inherent with each instrument. Most of the researchers (Childs and Scharrer, 1986 & 1988; Hawkins et al., 1989; Childs et al., 1989, Childs et al., 1990a; Childs and Ramsey, 1991; Childs et al., 1991; Childs and Kleyhans, 1992; Kurtin et al., 1993; Alexander et al. 1995; Childs and Gansle, 1996; Nielsen et al., 2001) used the method described by Holman (1978) to estimate the uncertainty in rotor dynamic parameters. The method is briefly stated as follows. Let the results \( R \) (e.g. rotor dynamic parameters) is a given function of the independent variables \( x_1, x_2, \ldots, x_n \) (e.g. rotor speed, inlet pressure, pressure drop, diameter, length, clearance, temperature, force, excitation frequency, displacement, acceleration, etc.). Thus,

\[
R = R(x_1, x_2, \ldots, x_n)
\]  

(14.83)

Let \( w_R \) be the uncertainty in the result and \( w_1, w_2, \ldots, w_n \) be the uncertainties in the independent variables. Then the uncertainty in the result is given as
\[ w_R = \left[ \left( \frac{\partial R}{\partial x_1} w_1 \right)^2 + \left( \frac{\partial R}{\partial x_2} w_2 \right)^2 + \cdots + \left( \frac{\partial R}{\partial x_n} w_n \right)^2 \right]^{1/2} \] (14.84)

with

\[ \frac{\partial R}{\partial x_1} = \frac{R(x_1 + \Delta x_1) - R(x_1)}{\Delta x_1}; \quad \frac{\partial R}{\partial x_2} = \frac{R(x_2 + \Delta x_2) - R(x_2)}{\Delta x_2}; \cdots \] (14.85)

where \( \Delta x_1, \Delta x_2, \cdots, \Delta x_n \) are the small perturbations of the independent variables. It should be noted that the uncertainty propagation in the results \( w_R \) predicted by equation (14.84) depends on the squares of the uncertainties in the independent variables \( w_n \). This means that if the uncertainty in one variable is significantly larger than the uncertainties in the other variables, then it is the largest uncertainty that predominates and other may probably be negligible. The relative magnitude of uncertainties is evident when one considers the design of an experiment, procurement of instrumentations, etc. Most of the researchers found effect of uncertainty measurement in the force, excitation frequency, and displacement measurements on the stiffness, damping and added mass coefficients.

According to Childs and Ramsey (1991), the principal source of uncertainty in the resultant force measurement was the acceleration measurement for the stator. Although more sensitive accelerometers are available, they cannot generally be used when testing honeycomb seals, because high-frequency acceleration spikes were frequently seen with these seals, presumably because of Helmholtz acoustic excitation of honeycomb cavities. Kanemori and Iwatsubo (1992), Kaneko et al. (1998, 2003) and Childs and Fayolle (1999) carried out uncertainty analysis using methods established by ANSI/ASME (1986). Childs and Wade (2004) obtained uncertainty of RDPs for variation of excitation frequency. They calculated the test uncertainty at each frequency as the square root of the sum of squares of baseline uncertainty and seal test uncertainty at each frequency.

14.13 General Remarks and Future Directions on Bearings

This paper has given a review of the identification procedures applied to bearing parameter estimation. The main emphasis has been to summarise the existing experimental techniques for acquiring measurement data from the rotor-bearing test rigs, theoretical procedures to extract the relevant bearing parameters and to estimate associated uncertainties. It is felt that both the experimental and theoretical aspects are important since most of the state of the art procedures require matrix inversion and if the experimental data acquired are not sufficiently independent, then ill-conditioning may occur. The following are the outcomes of the present review.
14.13.1 General Remarks on Bearings

a. Amongst the various bearing parameter identification methods available most of them require the bearing to be tested in isolation and very few to be tested as a rotor-bearing system where the shaft could be treated rigid or flexible.

b. For the case the bearing is tested in isolation, it is easy to control the operating conditions when journal-sleeve configuration is considered in its inverted form. The basic assumption behind this inverted form is that bearing dynamic coefficients do not depend upon operating conditions so long as the Sommerfeld number is kept constant.

c. It is well established that available theoretical rotor models are sufficiently accurate to assume to be known (when the rotor geometry and material properties are known or by using modal testing methods in a free-free end conditions of the rotor) in flexible rotor-bearing systems during the bearing dynamic parameter identification.

d. In general the coefficients derived from static load testing are extremely sensitive to measurement errors and the sensitivity of dynamically derived coefficients is more difficult to assess.

e. Frequency domain methods appear to be better in terms of the amount of data to be handled/stored, consequently the time required to process them during identification process, especially for the case of full-scale/actual machines. Moreover, the signal to noise ratio is found to be better for frequency domain methods.

f. Modal testing methods (Ewins, 1984) have been applied with some modification to rotor-bearing systems (Nordmann, 1984; Muszynska, 1986; Rogers and Ewins, 1989; Lee, 1991; Jei and Kim, 1993; Zhang and Xie, 1996; and Peeters et al., 2001a & b). However, the identification of modal parameters of a rotor bearing system is difficult or at least time consuming, and, moreover, there still remains a difficulty in accomplishing modal testing on rotor-bearing systems during operation.

g. While measuring the synchronous (unbalance) response, measurements should be taken for both forward and backward whirling of the rotor this ensures the availability of all the modal information in measurements. Providing a provision for the external excitation frequency to be anti-synchronous (i.e. the excitation frequency, $\Omega$, equals to the minus of the rotor speed, $\omega$) can do this.

h. For the case of isotropic bearings (for example rolling element bearings) where the journal orbits often becomes nearly circular for most of the operating speed ranges except near the resonances and this leads to ill-conditioning of the regression matrix. This could be well-conditioned by rotating the rotor both in the clockwise and anticlockwise directions and taking measurements. For the case where the dynamic characteristics changes (for example fluid-film bearings) with the direction of rotation of the rotor an external provision for the external excitation frequency to be anti-synchronous with respect to the rotation of the rotor could do it.
i. Corresponding to different sets of data the orbit of the journal should be as far as possible elliptical and it should change both in the magnitude and orientation of major (or minor) axis of the orbit. This ensures measurements to be independent.

j. In almost all available papers interactions between the bearings in rotor-bearing systems has not been taken into account. This assumption makes the bearing dynamic stiffness matrix banded and hence it is possible to identify the individual bearings separately.

k. When fluid film pressure distribution is used to calculate force transmitted through the bearing, care should be taken while estimating the bearing dynamic coefficients that this force is not equal to the external force applied to the bearing bush or the shaft, especially for higher excitation frequency.

l. From the governing dynamic equations effectively four equations are obtained to solve for at most eight unknown dynamic coefficients (either damping and effective stiffness or damping and effective added-mass) by using at least two independent force-response data for the case when bearing dynamic parameters are external excitation frequency dependent. Identification of all twelve coefficients by using at least three independent force-response data in a linear least squares fit is not possible. However, with the engineering assumption by using force-response data at minimum of three frequency points close to the frequency of rotation could be used to estimate added-mass, damping and stiffness coefficients of bearings.

m. With the assumption that the added-mass, damping and stiffness coefficients are independent of the external excitation frequency, by using force-response data at minimum of three frequency points could be used to estimate added-mass, damping and stiffness coefficients of bearings. Alternatively, with the above assumption since the stiffness force is independent of the external excitation frequency, the damping force is proportional to the external excitation frequency and the added-mass force (inertia) is proportional to the square of the external excitation frequency. For a given equilibrium position if the complex dynamic stiffness of the bearing is available for various excitation frequencies, then plots could be plotted between real part of dynamic stiffness versus square of external excitation frequency and imaginary part of the dynamic stiffness versus external excitation frequency. The intercept of the first plot with vertical axis at zero external excitation frequency will give the stiffness coefficients and slope of the line will give the added-mass coefficient. From second plot the slope of the line will give the damping coefficient. This method of identifying all twelve bearing dynamic coefficients by using minimum of three frequency points force-response data is not possible while using run-down/up responses. Since for this case frequency of excitation will be equal to the frequency of rotation of the rotor so in regression equation added-mass and stiffness coefficients terms will be linearly dependent and this leads to regression matrix to be singular.
n. Very few papers are available on rolling element bearings dynamic parameter estimation although papers are in abundance for its condition monitoring (Tandon and Choudhury, 1999). This may be because they are relatively stiff with little damping characteristics along with the complex structure of bearings and the related dynamics. This leads to difficulties in accurate mathematical modelling and performing meaningful measurements during normal operation to be used for estimating physical meaningful dynamic parameters. Whereas, condition monitoring methods generally rely on feature extraction methods for example statistical methods (i.e. using root-mean-square, standard deviation, skewness and kurtosis), FFT (i.e. frequency components corresponding to integer multiples of rotational frequency and number of rolling elements) and neural networks.

o. While using impact tests (multi-frequency test) it is necessary to remove the imbalance response from the signal especially at higher speed of operation.

p. In multi-frequency tests (either by using exciter or impact hammer) it is assumed that the bearing dynamic coefficients are independent of frequency of excitation.

q. The error due to the noise can be minimised by averaging the frequency response.

r. There is a need to minimise the time required to capture the measurement data for the in situ determination of the parameters, moreover to reduce problems due to parameter drift.

s. It is necessary to be able to place some confidence bounds on the estimates.

t. The effect of fluid inertia or added-mass should be considered for high Reynolds number (with laminar flow) bearings.

u. The indirect assessment of the estimates can be performed by response/force reproduction/prediction, especially for independent measurements that have not been incorporated in the parameter estimation.

14.13.2 Future Directions on Bearings

a. There is a still need for experimental work in the field of rotor dynamics to study the influence of bearings and supports upon the rotor response, in particular for full-scale rotor systems.

b. New experiments should be devised and more effective use of the available data needs to be made especially with the inherent practical constraints for measurements and development of new identification techniques.

c. The validation of dynamic coefficients using data derived from actual machines in the actual operating environment is required.

d. The technique must be capable of use with the rotor-bearing system being run under normal operating conditions and in situ.
e. Synchronous unbalance response, which can easily be obtained from the run-down/up of large turbomachineries, should be exploited more for the identification of bearing dynamic parameters along with the estimation of unbalance.

f. The bearing dynamic parameter identification from the response information corresponding to unknown forces (sinusoidal/periodic/random) inherent to the system should be further investigated.

g. Since foundations are an integral part of turbomachines and very few investigations have been performed of this type of machinery. Moreover, present foundation models have not proven to be so successful. There is a need to develop new effective foundation models and the complete identification of the bearing and foundation model along with the estimation of unbalance from run-down/up data.

h. Although there are some developments, however, a stage in the development of the state of the art has been reached in which improvements and further investigations are needed, with a better understanding of fundamental behaviour of bearing. Additional theoretical studies and experimental investigation are required to consider the influence of thermal and elastic distortion, grooving arrangements, misalignment, cavitation and film reformation on the dynamic coefficients of bearings.

i. From the governing dynamic equations effectively, four equations are obtained to solve for at most eight unknown dynamic coefficients. Since static equilibrium locus provides four additional equations to solve for the four stiffness coefficients, there is need for further investigation to exploit both the static and dynamic methods in combination for bearing dynamic parameters identification.

j. There is a need to develop new mathematical models of bearings (some times referred as the problem of bearing identification) based on the experimentation to represent the behaviour of rotor-bearing systems.

k. Application of the directional time series method available for rotor bearing system modal parameter estimation (Lee et al., 1997a & b) to the identification of the bearing parameters.

l. In spite of the majority of the rotor-bearing dynamic behaviours can be explained by the linear dynamic coefficient model of bearings, there is a need to develop new effective non-linear bearing model and their identification (Garibaldi and Tomlinson, 1988; Stanway et al., 1988; Burrows et al., 1990; Tiwari, 2000) so as to use for predicting non-linear phenomena (for example jump and flutter phenomena). The application of advanced engineering methods (such as the Fokker-Planck equation (Dimentberg, 1988), Volterra and Wiener based technique (Schetzen, 1980)) for the experimental identification of non-linear dynamic parameters of rotor-bearing systems needs further investigation.
m. In spite of relatively stiff and light damping characteristics of rolling element bearings, they affect
the dynamics of rotors especially for high-speed applications (Jones, 1960; Harris, 1971; Gupta,
1984). As compared to fluid film bearings the dynamic characteristics of rolling element bearings
change drastically due to change in preloads (i.e. negative clearance) over a period of operations
due to wear and tear of bearings rolling contact surface and subsurface (While, 1979; Tiwari and
Vyas, 1995). There is a need to characterise dynamic properties accurately and on-line, especially
in practical situation where rolling element bearings (turbojets) operate in more variable operating
conditions as compared to fluid-film bearings (turbogenerators).

n. Interactions between the bearings should be investigated and appropriate bearing models should
be identified.

o. The dynamic bearing coefficient dependency on the external excitation frequency for a given
operating condition needs further investigation.

p. Experimental identification of dynamic coefficients of the thrust bearings is rare and needs special
attention.

q. Active (such as fluid film bearing with accumulator and magnetic bearing) and passive (such as
squeeze-film bearing) control bearings need to be developed and identified.

r. With the development of smart fluids (for example electro-rheological and magneto-rheological;
Stanway et al., 1996 and Sims et al., 1999) there is a need to develop and identify controllable
vibration damping devices.

s. Identification methods need to be developed for the distributed bearing stiffness and damping
coefficients (consistent stiffness and damping matrices (Craggs, 1993; Rao et al., 1996)).

t. In order to be useful in future for analysts, the identified dynamic parameters of bearings should
be documented in tabular form and/or equation coefficients for curve fits to the plots along with
well documentation of the operating conditions.

u. There is a need for standardisation of the data to be provided for publications in the field of
bearing dynamic parameter identification.

v. From the present state of the art methods of identification of dynamic bearing parameters a
comprehensive collection of data similar to Someya (1989) is expected in near future by joint
efforts of researchers in the field of the bearing identification.

w. There is a need for raw/processed measured data to be made available/exchange among the
researchers in the field of the bearing identification.
14.14 General Remarks and Future Directions on Seals

1. For the last three decades there has been increased concern over the seal induced instability and hence exponential increase in research on RDPs of seals. Theoretical and experimental estimation of RDPs of seals are much more complicated than for bearings.

2. The cross-coupled stiffness force (e.g. $k_{xy} \Delta y$) arises due to fluid rotation within the seal clearance and acts in opposite to the direct damping force (e.g. $c_{xx} \Delta \dot{x}$) to destabilize the rotor. Hence, to improve the rotor stability steps should be taken to reduce the net fluid rotation within the seal by reducing the cross-coupled stiffness.

3. Small direct stiffness coefficients can be undesirable from rotordynamics viewpoint, since it may decrease the rotor critical speed. Optimally tapered seals have significantly larger direct stiffness than straight seals.

4. Grooving on stator/rotor significantly reduces stiffness and damping effects, possibly as much as 80% reduction with wide and deep grooves. Having grooves on seal stator (teeth-on-stator seal) is rotordynamically more stable than having grooves on the rotor (teeth-on-rotor seal).

5. In labyrinth/honeycomb seals, leak flow rate is less than that in smooth annular seal. In brush seals, leakage is very less compared with labyrinth/honeycomb seals. The major improvement provided by these labyrinth/honeycomb and brush seals are a significant reduction in tangential flow velocity within the seal, which significantly reduces the destabilizing cross-coupled stiffness effect. However, the bristle motion characterized by circumferentially traveling waves occurs in the bristles leads to their premature wear-out. Labyrinth/honeycomb seals also yield a reduction in main stiffness coefficients.

6. Honeycomb seals are used to eliminate rotordynamic instabilities in compressor or turbine applications. The stability measured by the whirl frequency ratio improves as the pressure ratio increases. The stability is shown very sensitive to changes in cell dimensions with improved stability for the larger cell sizes.

7. Swirl brakes in the upstream seal, which reduce the inlet tangential velocity, could substantially reduce or eliminate the cross-coupled stiffness coefficient, $k_c$. For improving the efficiency and stability margin of the pumps, it is generally required that annular seals reduce $k_c$ and the leakage flow rate and increase in the direct-stiffness ($k_d$) and direct-damping coefficients ($c_d$).

8. Hybrid brush pocket damper seal is the combination of pocket damper and brush seals, which are mainly for high damping and low leakage, respectively. Therefore, this seal combines high damping with low leakage.

9. The geometry of rotary seals is quite complex and it leads to difficulty in modeling and analyzing seals by theoretical and computational methods. Analytical models of rotary seals
are multifaceted and still under improvement with new seals design. Analytical and computational analyses methods of seals are tedious and still under development.

10. Various geometric parameters of seals affect RDPs such as diameters, clearances and lengths of seals; hole diameters, density of holes and hole depths in hole-pattern seals and other dimensions defining different pattern (the honeycomb, the triangular, etc.) seals etc. There is a need to develop new mathematical models of seals based on experiments to represent the behavior of rotor-bearing-seal systems.

11. Available experimental data resources on RDPs of seals can be used for only qualitative comparisons since number of parameters affecting RDPs is large and in several cases all these parameters are not reported. Hence, there is a need for standardization of the data given in publications in the field of seal RDPs estimation. Moreover, for maximum usefulness to analysts, the identified RDPs of seals should be documented in tabular form and equation coefficients for curve fits given along with the documentation of operating conditions. There is a need for raw/processed measured data to be made available and exchanged among the researchers in the field.

12. Experimental estimation of RDPs of seals has been mainly obtained on dedicated test rigs under controlled excitations. The validation of the seals RDPs using data derived from actual machines in the actual operating environment (from excitations inherent in systems) is required. Synchronous unbalance response, which can easily be generated and are present inherently in any rotor systems, should be exploited more for the estimation of RDPs along with the estimation of residual unbalances.

13. Frequency-domain methods are preferable in terms of the quantity of data to be handled/stored. The signal-to-noise ratio is found to be better for frequency-domain methods. And hence estimation of RDPs of seals is more reliable in frequency domain. While using impact tests (multi-frequency tests) it is necessary to remove the unbalance response from the signal, especially at higher speeds of operation. In multi-frequency tests (either by using an exciter or impact hammer) it is assumed that the seal RDPs are independent of the frequency of excitation, which may not be true for all the cases.

14. Theoretical, computational and experimental error analysis of RDPs of seals should be presented as integral part of all the estimates.

15. There is still a need for experimental work in the field of rotor dynamics to study the influence of seals and supports upon the rotor response, in particular for full-scale rotor systems. New experiments should be devised and more effective use of the available data needs to be made, especially with the inherent practical constraints for measurements and development of new estimation techniques.
14.15 Conditioning of Regression Matrices for Simultaneous Estimation of the Residual Unbalance and Bearing Dynamic Parameters

The residual unbalance estimation in rotor-bearing system is an age-old problem. From the state of the art of the unbalance estimation, the unbalance can be obtained with fairly good accuracy (Kellenburger, 1972; Drechsler, 1980; Gnilka, 1983, Krodkiewski et al., 1994; Darlow, 1989). Now the trend in the unbalance estimation is to reduce the number of test runs required, especially for the application of large turbogenerators where the downtime is very expensive (Edwards et al., 2000).

Bearing dynamic parameters play an important role in the prediction of the dynamic behaviour of rotor-bearing systems. Due to the difficulty in obtaining actual test conditions the well-developed theoretical analysis to obtain bearing dynamic parameters by using the Reynolds equation fails to give satisfactory results (Lund, 1980). This forces designers to look for the experimentally obtained dynamic bearing parameters. Exhaustive work has been done on the experimental identification of the eight linearised bearing dynamic parameters (Ramsden, 1967-68; Goodwin, 1991; Swanson and Kirk, 1997; Tiwari et al., 2004; Tiwari et al. 2005). Most of the work is based on the dynamic force, which is given to the system, and corresponding response is measured. At least two independent force-response relationships are used to obtain bearing dynamic parameters. Researchers have reported that often the regression matrix of the estimation equation becomes ill-conditioned and this leads to scattering of the estimated bearing dynamic parameters (Tiwari et al., 2002).

Advances in the sensor technology and the increase in computing power in terms of the amount of data can be collected/handled and the speed at which it can be processed leads to the development of methods that could be able to estimate residual unbalance along with bearing dynamic parameters simultaneously (Chen and Lee, 1995; Lee and Shih, 1996; Tiwari and Vyas, 1997, Sinha et al., 2002). These methods generally estimate the residual unbalance accurately but the estimation of bearing dynamic parameters suffers from scattering due to the ill-conditioning of the regression matrix of the estimation equation (Tiwari et al., 2002; Sinha et al., 2002; Tiwari, 2005).

In the present section a general identification algorithm to obtain unknown parameters of the multi-degree of freedom rotor-bearing systems with gyroscopic effects is presented. Lots of researches were carried out on bearing dynamic parameters estimation (Goodwin, 1991) by treating the rotor as rigid. Because of high level of uncertainty in the bearing dynamic parameters estimates, still researchers have been concentrating on rigid rotor models, while making dedicated experimental set-up for the bearing dynamic parameters estimation (Tiwari et al., 2004; Tiwari et al. 2005). Residual unbalances gives rise to some amount of error in estimates of bearing dynamic parameters. The present paper
addresses the issue of improving bearing dynamic parameters estimation by simultaneously estimating the unknown residual unbalances. Gyroscopic couples are significant when the rotor speed is very high and the diametral mass moment of inertia is large. In laboratory experimental set-up researchers do go for high speed, however, the diametral mass moment of inertia is not significant. Moreover, gyroscopic effects give rise to rotational degree of freedom in equations of motion and accurate measurement of the same is still a challenging problem (Dharmaraju et al., 2004) (unless they could be eliminated by condensation methods). The effect of gyroscopic effects is ignored for the present case illustrations. In the present work an analysis of the ill-conditioning of the regression matrix, while estimating both the residual unbalance and bearing dynamic parameters of rotor-bearing systems, is presented. The conditions in which ill-conditioning can occur have been also examined. Theoretical and experimental methods are suggested, as how to improve the condition the regression matrix, in the form of different formulations and types of measurements, respectively. The improvement in the conditioning of the regression matrix is illustrated by using simulated examples. The effect of noise in the measurement responses is also tested.

Equations of Motion and Response

For a rigid rotor supported on two identical flexible bearings with an independent unbalance excitation unit (Tiwari et al., 2004), a schematic diagram is shown in Figures 14.35 and 14.36, equations of motion can be written as

\[
m \ddot{x} + c_x \dot{x} + c_y \dot{y} + k_x x + k_y y = \omega^2 U_x^{\text{Res}} e^{j\Omega t} + \omega^2 U_x^{\text{Tri}} e^{j\Omega t}
\]

and

\[
m \ddot{y} + c_x \dot{x} + c_y \dot{y} + k_x x + k_y y = \omega^2 U_y^{\text{Res}} e^{j\Omega t} + \omega^2 U_y^{\text{Tri}} e^{j\Omega t}
\]

where \( m \) is the rotor mass per bearing, \( c_{ij} \) and \( k_{ij} \) (with \( i, j = x, y \)) are the linearised damping and stiffness coefficients of the bearing, \( U \) is the unbalance (i.e. multiplication of the unbalance mass and its eccentricity; in general it is a complex quantity and it contains the magnitude and phase information), \( x \) and \( y \) are the rotor linear displacements in the vertical and horizontal directions respectively, \( \omega \) is the rotational speed of the rotor, \( \Omega \) is the frequency of excitation from the auxiliary unbalance excitation unit, \( t \) is the time instant and \( j = \sqrt{-1} \). Superscripts: \( \text{Res} \) and \( \text{Tri} \) represent the residual and trial unbalance masses respectively.
Taking the solution of equations (14.86) and (14.87) in the following form

\[ x = X^r e^{i\omega t} + X^i e^{i\Omega t} \]  \hspace{1cm} (14.88)

and

\[ y = Y^r e^{i\omega t} + Y^i e^{i\Omega t} \]  \hspace{1cm} (14.89)

where the \( X \) and \( Y \) contain the magnitude and phase information of the respective displacements and in general they are complex quantities (i.e. \( X = X_r + jX_i \), etc.; where subscripts \( r \) and \( i \) represent real and imaginary, respectively). Since these equations of motion are linear in nature, on substituting equations (14.88) and (14.89) into equations (14.86) and (14.87), we get
\[
\begin{bmatrix}
(k_{xx} - m\omega^2 + j\omega \alpha_{xx}) & (k_{xy} + j\omega \alpha_{xy}) \\
(k_{yx} + j\omega \alpha_{yx}) & (k_{yy} - m\omega^2 + j\omega \alpha_{yy})
\end{bmatrix}
\begin{bmatrix}
X_{Res}^R \\
Y_{Res}^R
\end{bmatrix}
= \omega^2 \begin{bmatrix}
U_{x}^{Res} \\
U_{y}^{Res}
\end{bmatrix}
\tag{14.90}
\]

and
\[
\begin{bmatrix}
(k_{xx} - m\Omega^2 + j\Omega \alpha_{xx}) & (k_{xy} + j\Omega \alpha_{xy}) \\
(k_{yx} + j\Omega \alpha_{yx}) & (k_{yy} - m\Omega^2 + j\Omega \alpha_{yy})
\end{bmatrix}
\begin{bmatrix}
X_{Tri}^R \\
Y_{Tri}^R
\end{bmatrix}
= \Omega^2 \begin{bmatrix}
U_{x}^{Tri} \\
U_{y}^{Tri}
\end{bmatrix}
\tag{14.91}
\]

For known unbalance information (both the residual and trial unbalances) and rotor-bearing parameters, equations (14.90) and (14.91) can be used to obtain the displacement amplitude and phase components. Equations (14.90) and (14.91) can be combined as
\[
\begin{bmatrix}
D_{xx}(\omega) - m\omega^2 & D_{xy}(\omega) & 0 & 0 \\
D_{yx}(\omega) & D_{yy}(\omega) - m\omega^2 & 0 & 0 \\
0 & 0 & D_{xx}(\Omega) - m\Omega^2 & D_{xy}(\Omega) \\
0 & 0 & D_{yx}(\Omega) & D_{yy}(\Omega) - m\Omega^2
\end{bmatrix}
\begin{bmatrix}
X_{Res}^R \\
Y_{Res}^R \\
X_{Tri}^R \\
Y_{Tri}^R
\end{bmatrix}
= \begin{bmatrix}
\omega^2 U_{x}^{Res} \\
\omega^2 U_{y}^{Res} \\
\Omega^2 U_{x}^{Tri} \\
\Omega^2 U_{y}^{Tri}
\end{bmatrix}
\tag{14.92}
\]

with
\[
D_{ij}(\omega) = k_{ij} + j\omega \alpha_{ij} \quad \text{and} \quad D_{ij}(\Omega) = k_{ij} + j\Omega \alpha_{ij} \quad i, j = x, y \tag{14.93}
\]

For developing the identification algorithm for the simultaneous estimation of the residual unbalance and bearing dynamic parameters, the frequency domain equations of motion in form of equation (14.92) is used. For the case when the auxiliary excitation unit is not used, however, the trial mass are attached to the rotor itself, then the solution of equations (14.86) and (14.87) could be assumed in the following form
\[
x = X e^{ix} \quad \text{and} \quad y = Y e^{iy} \tag{14.94}
\]

where the \(X\) and \(Y\) contain the magnitude and phase information of displacements and in general they are complex quantities. On substituting equation (14.94) into equations (14.86) and (14.87), we get
\[
\begin{bmatrix}
D_{xx}(\omega) - m\omega^2 & D_{xy}(\omega) \\
D_{yx}(\omega) & D_{yy}(\omega) - m\omega^2
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \omega^2 \begin{bmatrix}
U_{x}^{Res} + U_{x}^{Tri} \\
U_{y}^{Res} + U_{y}^{Tri}
\end{bmatrix}
\tag{14.95}
\]

For the known unbalance information (i.e. both the residual and trial) and rotor-bearing parameters, equation (14.95) can be used to obtain the displacement amplitude and phase at any spin speed of the
rotor. For developing the identification algorithm for simultaneous estimation of the residual unbalance and bearing dynamic parameters, the frequency domain equations of motion in the form of equation (14.95) will also be used.

Simultaneous Estimation of the Residual Unbalance and Bearing Dynamic Parameters

The equation (14.95) can be rearranged such that all the unknown quantities (i.e. the residual unbalance and bearing dynamic parameters) are in the left hand side of the equation and the right hand side contains all the known information, in the following form

\[
D_X(\omega)X + D_Y(\omega)Y - \omega\dot{U}_s^{Res} = \omega\dot{U}_X^{Tri} + m\omega^2 X
\]  
(14.96)

and

\[
D_X(\omega)X + D_Y(\omega)Y - \omega\dot{U}_s^{Res} = \omega\dot{U}_Y^{Tri} + m\omega^2 Y
\]  
(14.97)

Noting that the unbalance force components (both for the residual and trial unbalances) in two orthogonal directions are related as

\[
U_y = \bar{\omega} j U_x
\]  
(14.98)

where the first sign (i.e. negative) is for the case when the direction of rotation of the rotor is in accordance with the positive axis direction conventions and the second sign (positive) is for the case when the direction of rotation of the rotor is in the negative direction of the right hand axis conventions. Noting equation (14.98), equations (14.96) and (14.97) can be combined as

\[
\begin{bmatrix}
X \\ Y
\end{bmatrix}
\begin{bmatrix}
D_X(\omega) & 0 & -\omega^2 \\
0 & D_Y(\omega) & j\omega^2 \\
0 & X & Y \\
0 & 0 & U_s^{Res}
\end{bmatrix}
\begin{bmatrix}
D_X(\omega) \\ D_Y(\omega) \\ D_X(\omega) \\ D_Y(\omega) \\ U_s^{Res}
\end{bmatrix}
= \begin{bmatrix}
\omega\dot{U}_X^{Tri} + m\omega^2 X \\ \omega\dot{U}_Y^{Tri} + m\omega^2 Y \\
\end{bmatrix}
\]  
(14.99)

For the case when the auxiliary unbalance excitation unit is used, it is convenient to express the estimation equation from equation (14.92) in the following form

\[
\begin{bmatrix}
A_x(\omega) \\ A_z(\Omega)
\end{bmatrix}
\begin{bmatrix}
b
\end{bmatrix}
= \begin{bmatrix}
q_1(\omega) \\ q_2(\Omega)
\end{bmatrix}
\]  
(14.100)
In this section two form of the estimation equation have been developed i.e. equations (14.99) and (14.100). These are the required form of the estimation equation in which all the unknowns i.e. the residual unbalance and bearing dynamic parameters are stacked in a column vector. These equations can be used to obtain these unknown parameters with the knowledge of the response, the rotational frequency of the rotor/excitation, the rotor mass and the trial unbalance information. From both forms of estimation equations, it can be seen that the number of unknowns are more than the number of equations, which is a case of underdetermined system of linear simultaneous equations. The entire unknown can be obtained with the help of sets of independent force-response measurements such that the number of equations increases at least equal to or more than unknowns. However, the consideration of the condition of the regression matrix to be inverted has important role in obtaining the better estimate of parameters.

Condition of the Regression Matrix

The present section deals with various possibilities of measurements of independent responses while keeping the speed of the rotor constant. The following three types of measurements are discussed (a) with sets of trial unbalances (b) for the above case with rotating the rotor in clockwise and counter
clockwise direction, alternatively and (c) rotor with an independent unbalance excitation unit. For each of the above cases the condition of the regression matrix is also discussed.

**Method 1:** By taking measurements with three different sets of trial masses (one without trial mass and other with two different trial masses; it is assumed that the rotor has always some amount of residual unbalance) at the same rotational speed of the rotor, six equations can be obtained to solve for five unknowns. Noting equation (14.99), the form of regression equation would be

\[
\begin{bmatrix}
A(\omega) \equiv [b(\omega)] = [q(\omega)] \\
\end{bmatrix} \tag{14.106}
\]

with

\[
A(\omega) = \begin{bmatrix}
X_1 & Y_1 & 0 & 0 & -\omega^2 \\
0 & 0 & X_1 & Y_1 & j\omega^2 \\
X_2 & Y_2 & 0 & 0 & -\omega^2 \\
0 & 0 & X_2 & Y_2 & j\omega^2 \\
X_3 & Y_3 & 0 & 0 & -\omega^2 \\
0 & 0 & X_3 & Y_3 & j\omega^2 \\
\end{bmatrix} \tag{14.107}
\]

\[
b(\omega) = \begin{bmatrix}
D_{x_1}(\omega) & D_{y_1}(\omega) & D_{x_2}(\omega) & D_{y_2}(\omega) & U^{\text{Res}}_x \end{bmatrix}^T \tag{14.108}
\]

\[
q(\omega) = \omega^2 \begin{bmatrix}
mX_1 & mY_1 & (U^{\text{Tr} x} + mX_2) & (U^{\text{Tr} y} + mY_2) & (U^{\text{Tr} z} + mX_3) & (U^{\text{Tr} z} + mY_3) \end{bmatrix}^T \tag{14.109}
\]

It should be observed that the vector, \(b(\omega)\), is dependent on the rotor speed. Equation (14.106) is a standard form of the regression equation and it can be used to obtain the residual unbalance and bearing dynamic parameters with the help of the least squares fit from the following equation

\[
b(\omega) = (A(\omega)^T A(\omega))^{-1} A(\omega)^T q(\omega) \tag{14.110}
\]

It should be noted that the solution of the above equation requires inversion of the square matrix \((A(\omega)^T A(\omega))\). The matrix condition can be judged by obtaining the condition number of the matrix. The condition number of a matrix (with respect to inversion) measures the sensitivity of the solution of a system of linear equations to errors in the data. It gives an indication of the accuracy of the results from matrix inversion and the linear equation solution. Values of the condition number near 1 indicate a well-conditioned matrix. The 2-norm condition number, which is used in the present case, is defined as the ratio of the largest singular value of the regression matrix to the smallest (Golub and
Van Loan, 1996). It has been found that the matrix \( \begin{bmatrix} A(\omega) \end{bmatrix}^T \begin{bmatrix} A(\omega) \end{bmatrix} \) is highly ill-conditioned (however, masked by measurement noise while actual responses are processed) since the determinant of the matrix is nearly zero i.e.

\[
\left| \begin{bmatrix} A(\omega) \end{bmatrix}^T \begin{bmatrix} A(\omega) \end{bmatrix} \right| = 0
\] (14.111)

This poses difficulty in identifying the required parameters uniquely. It has been found that even while increasing the number of trial runs to four or even more, does not help in improving the condition of the above matrix. Moreover, as it could be seen from equation (14.107) by scaling of the last column (i.e. by \( 1/\omega^2 \)) the condition of the regression matrix may be improved. However, the condition of equation (14.111) holds after the scaling also.

**Method 2:** In this method measurements are taken by rotating the rotor first in the clockwise direction and then in the counter clockwise direction. It is assumed that bearing parameters do not change with the change in the sense of rotation (i.e. clockwise or counter clockwise). For symmetric and most commonly used bearings e.g. cylindrical fluid film bearings, rolling element bearings, etc., it may be the case. This approach is applied to the method described above. The measurements with only one trial unbalance is sufficient and the form of equations would be

\[
\begin{bmatrix} A(\omega) \end{bmatrix} \begin{bmatrix} b(\omega) \end{bmatrix} = \begin{bmatrix} q(\omega) \end{bmatrix}
\] (14.112)

with

\[
A(\omega) = \begin{bmatrix}
X_1 & Y_1 & 0 & 0 & -\omega^2 \\
0 & 0 & X_1 & Y_1 & j\omega^2 \\
X_2 & Y_2 & 0 & 0 & -\omega^2 \\
0 & 0 & X_2 & Y_2 & -j\omega^2 \\
X_3 & Y_3 & 0 & 0 & -\omega^2 \\
0 & 0 & X_3 & Y_3 & j\omega^2
\end{bmatrix}
\] (14.113)

\[
b(\omega) = \begin{bmatrix} D_{xx}(\omega) & D_{xy}(\omega) & D_{yx}(\omega) & D_{yy}(\omega) & U_x^{Re} \end{bmatrix}^T
\] (14.114)

\[
q(\omega) = \omega^2 \begin{bmatrix} mX_1 & mY_1 & mX_2 & mY_2 & U_x^{Tr} + mX_3 & U_y^{Tr} + mY_3 \end{bmatrix}^T
\] (14.115)
where \((X_1, Y_1)\) are measurements when the rotor having only the residual unbalance is rotated in the positive axis direction, \((X_2, Y_2)\) are measurements when the rotor with the residual unbalance is rotated in the negative axis direction and \((X_3, Y_3)\) are measurements when a trial mass is also added on the rotor and it is rotated in the positive axis direction. On comparing the matrix \([A(\omega)]\) of the equation (14.113) with that of equation (14.107), it can be seen that the matrix element \((4, 5)\) gets its sign changed because of the negative axis direction of rotation of the rotor (refer equation (14.98)). The condition number of the matrix \([A(\omega)]^T [A(\omega)]\) improves drastically. The fourth measurement can also be incorporated with the trial unbalance when the rotor is rotated in the opposite direction. More measurements can be also incorporated with different trial unbalances.

Method 3: For the case when the rotor cannot be rotated in either direction (i.e. clock wise and anti-clock wise) or the bearing parameters change with the direction of rotation of the rotor, an independent unbalance excitation unit is generally used. For such case it is assumed that rotor is always rotated in the positive axis direction. The condition of the regression matrix for identification of the bearing dynamic parameters improves when such arrangement is used especially for the case when two excitation frequencies are anti-synchronous (Tiwari et al., 2002). The three independent measurements could be obtained, first corresponding to the rotor rotational frequency and two measurements corresponding to two excitation frequencies \((\Omega_1, -\Omega_2)\) i.e. the negative sign indicate the sense of rotation is opposite to the positive axis direction. It is assumed that for all the three measurements the rotor speed remains constant. From equation (14.100) the regression equation takes the following form

\[
\begin{bmatrix}
\{A(\omega)\}
\end{bmatrix}
\begin{bmatrix}
b
\end{bmatrix}=
\begin{bmatrix}
\{q(\omega)\}
\end{bmatrix}
\tag{14.116}
\]

with

\[
\begin{bmatrix}
\{A(\omega)\}=
\begin{bmatrix}
A_1(\omega) & A_2(\Omega_1) & A_2(-\Omega_2)
\end{bmatrix}^T
\end{bmatrix}
\tag{14.117}
\]

\[
\begin{bmatrix}
\{q(\omega)\}=
\begin{bmatrix}
q_1(\omega) & q_2(\Omega_1) & q_2(-\Omega_2)
\end{bmatrix}^T
\end{bmatrix}
\tag{14.118}
\]

\[
A_2(-\Omega_2) = A_2(\Omega_2)
\tag{14.119}
\]
where matrices $[A_1(\omega)]$ and $[A_2(\Omega)]$ are defined in equations (14.101) and (14.102) respectively, and vectors $\{q_1(\omega)\}$ and $\{q_2(\Omega)\}$ are defined by equations (14.104) and (14.105) respectively. The vector $\{b\}$, as defined in equation (14.114), contains residual unbalances in the last two rows. Noting equation (14.98), accordingly matrices $[A_1(\omega)]$ and $[A_2(\Omega)]$ are affected only in the last two columns when the direction of residual unbalance changes. Since for the present case the direction of the residual unbalance does not change, equation (14.119) holds good. Moreover, it can be observed that equation (14.102) has last two columns as zero entries. With the above arrangement it can be seen that the regression matrix remains ill-conditioned since on rotating the trial mass in two different directions does not affect the regression matrix as the trial unbalance are contained in the vector $\{q\}$ (i.e. equation (14.105)).

**Example 14.7** A rotor-bearing system as shown in Figure 14.18 is considered for the numerical simulation to illustrate the foregoing section methods of simultaneous estimation of unbalance and bearing dynamic parameters. The rotor mass is 0.447 kg. Both the bearings are assumed to be identical and its dynamic properties are given in Table 14.1. Residual and trial unbalance magnitudes and angular positions are tabulated in Table 14.2.

### Table 14.1 Bearing dynamic parameters

<table>
<thead>
<tr>
<th>Bearing Parameters</th>
<th>$k_{xx}$</th>
<th>$k_{xy}$</th>
<th>$k_{yx}$</th>
<th>$k_{yy}$</th>
<th>$c_{xx}$</th>
<th>$c_{xy}$</th>
<th>$c_{yx}$</th>
<th>$c_{yy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>16788.0</td>
<td>1000.0</td>
<td>1000.0</td>
<td>18592.0</td>
<td>6.0</td>
<td>0.0</td>
<td>0.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

### Table 14.2 Residual and trial unbalances

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Type of unbalance</th>
<th>Magnitude (kg)</th>
<th>Radius (m)</th>
<th>Angular location (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Residual unbalance</td>
<td>0.001</td>
<td>0.03</td>
<td>0.0</td>
</tr>
<tr>
<td>2.</td>
<td>Trial unbalance 1</td>
<td>0.002</td>
<td>0.03</td>
<td>90</td>
</tr>
<tr>
<td>3.</td>
<td>Trial unbalance 2</td>
<td>0.003</td>
<td>0.03</td>
<td>60</td>
</tr>
</tbody>
</table>

*Solution:* The rotor response is generated at different unbalance configurations at a particular rotor speed by using equation (14.95). The simulated response is fed into the identification algorithm of Method 1 (i.e. equation (14.106)) to estimate the residual unbalance and bearing dynamic parameters. The estimated parameters are tabulated in Table 14.3 along with the condition number of the regression matrix and results suggest that it suffers from ill-conditioning. However, it should be noted
that the residual unbalance could be estimated accurately and this has been reported in Sinha et al. (2002). To improve (i.e. to reduce) the condition number of the regression matrix (i.e. equation (14.107)) the last column is scaled by a factor of $\omega^2 \times 10^{-4}$, so that the last column entries are of the same order as that of the rest. The corresponding condition number of the matrix is also tabulated in Table 14.3. Drastic improvement in the condition number occurred due to the scaling, however, for the present case no improvement in the estimated has been found. The similar identification exercise is performed by contaminating simulated responses by the random noise of 1% or 5%. The corresponding estimated parameters and the condition number of the regression matrix are also tabulated in Table 14.3 and most of the bearing dynamic parameters have ill-conditioning effects.

By rotating the rotor alternatively in the clockwise or counter clockwise direction the rotor response is generated at different unbalance configurations at a particular rotor speed by using equation (14.95). The simulated response is fed into the identification algorithm of Method 2 (i.e. equation (14.112)) to estimate the residual unbalance and bearing dynamic parameters. The estimated parameters are tabulated in Table 14.3 and results suggest that the regression equation is now well conditioned. The corresponding condition number of regression matrix (i.e. equation (14.113)) is also tabulated in Table 14.3 and as compared to Method 1 the condition number of the regression matrix is much lower. For the present case, however, it is observed that there is no improvement in the estimates of the parameters. The same exercise is performed by contaminating simulated responses by the random noise of 1% or 5%. The corresponding estimated parameters and condition numbers are also tabulated in Table 14.3 and they suggest the robustness of the present algorithm against measurement noise.

With the auxiliary unbalance unit (i.e. Method 3) the identification algorithm suffers from ill-conditioning similar to the Method 1 and result trends are identical to the Method 1.
<table>
<thead>
<tr>
<th>Parameters chosen for simulation of responses</th>
<th>Parameters estimated by the identification for the unbalance and bearings dynamic parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Method 1</td>
</tr>
<tr>
<td></td>
<td>Three runs without noise</td>
</tr>
<tr>
<td>( k_{xx} ) (N/m)</td>
<td>16788.0</td>
</tr>
<tr>
<td>( k_{yy} ) (N/m)</td>
<td>1000.0</td>
</tr>
<tr>
<td>( k_{yx} ) (N/m)</td>
<td>1000.0</td>
</tr>
<tr>
<td>( k_{yy} ) (N/m)</td>
<td>18592.0</td>
</tr>
<tr>
<td>( c_{xx} ) (N-s/m)</td>
<td>6.0</td>
</tr>
<tr>
<td>( c_{yy} ) (N-s/m)</td>
<td>0.0</td>
</tr>
<tr>
<td>( c_{yx} ) (N-s/m)</td>
<td>0.0</td>
</tr>
<tr>
<td>( c_{yy} ) (N-s/m)</td>
<td>6.0</td>
</tr>
<tr>
<td>Residual unbalance (kg-m)</td>
<td>3.0E-5</td>
</tr>
<tr>
<td>Residual unbalance phase (deg.)</td>
<td>0.0</td>
</tr>
<tr>
<td>Condition number of regression matrix without scaling</td>
<td>-</td>
</tr>
<tr>
<td>Condition number of the regression matrix after column scaling</td>
<td>-</td>
</tr>
</tbody>
</table>
High-speed rotating machineries, such as steam and gas turbines, compressors, blowers and fans, find wide applications in engineering systems. The danger of residual unbalances in such machineries attracted attention of researchers during quite early days (Rankine, 1869; Dunkerley, 1894; Jeffcott, 1919). From the state of the art, methods of balancing can be categorized into two groups; the influence coefficient method, which only requires the assumption of linearity of both the machine and measuring system, and modal balancing which in addition, requires knowledge of the modal properties of the machine. Influence coefficient method requires less a priori knowledge of the system and techniques have been well developed to make optimum use of redundant information (Drechsler, 1980). The approach has a significant disadvantage of requiring a number of test runs on site. Modal approaches require fewer test runs, Gnielka (1983) used prior knowledge of the mode shapes and modal masses and compared results to those from a numerical model of the machine. The work of Krodkiewski et al. (1994) has similar requirements and seeks to detect changes in unbalance from running data. Both these approaches place reliance on the numerical model. Numerical models of rotating machinery have been used to great effect over a number of years (McCloskey and Adams, 1992), and their accuracy and range of effectiveness have been steadily developing. Traditional turbo generators balancing techniques require at least two run-downs, with and without the use of trial weights respectively, to enable the machine's state of unbalance to be accurately calculated (Parkinson, 1991). Lees and Friswell (1997) presented a method to evaluate state of unbalance of rotating machine utilising the measured pedestal vibration. Subsequently, Edwards et al. (2000) presented the experimental verification of the method (Lees and Friswell, 1997) to evaluate the state of unbalance of a rotating machine. From the state of the art of the unbalance estimation procedure, the unbalance could be obtained with fairly good accuracy. Now the trend in the unbalance estimation is to reduce the number of test runs required especially for the application of large turbogenerators where the downtime is very expensive.

Rotating machineries are supported by bearings, which play a vital role in determining the behaviour of the rotating system under the action of dynamic loads. One of the most important factors governing the vibration characteristics of rotating machinery is bearing dynamic parameters. The influence of bearing dynamic characteristics on the performance of the rotor-bearing system was also recognized for a long time. One of the earliest attempts to model a journal bearing was reported by Stodola (1925) and Hummel (1926). They represented the fluid-film of bearings as a simple spring support, but their model was incapable of accounting for the observed finite amplitude of oscillation of a shaft operating at a critical speed. Concurrently, Newkirk (1924) and Newkirk and Taylor (1925) described the phenomenon of bearing induced instability, which they called oil whip, and it soon occurred to several investigators that the problem of rotor stability could be related to the properties of the bearing.
dynamic coefficients. Although the importance of rotor support dynamic stiffness is generally well recognized by the design engineer it is often the case that theoretical models available for predicting it are insufficiently accurate, or are accurate only in very specific cases. Moreover, the stiffness and damping characteristics are greatly dependent on many physical and mechanical parameters such as the lubricant temperature, the bearing clearance and load, the journal speed and the machine misalignment in the system and these are difficult to obtain accurately in actual test conditions. The uncertainties about machine parameters can make inaccurate results, obtained with the best theoretical methods aimed to study the behaviour of fluid-film journal bearings. Owing to this, it can be very useful to determine the bearing dynamic stiffness by means of identification methods based on experimental data and machine models. It is for this reason that designers of high-speed rotating machinery mostly rely on experimentally estimated values of bearing stiffness and damping coefficients in their calculations.

Several time domain and frequency domain techniques have been developed for experimental estimation of bearing dynamic coefficients. Many works have dealt with identification of bearing dynamic coefficients and rotor-bearing system parameters using the impulse, step change in force, and synchronous and non-synchronous unbalance excitation techniques. Ramsden (1967-68) was the first to review the papers on the experimentally obtained journal bearing dynamic characteristics. In mid seventies Dowson and Taylor (1980) conducted a survey in the field of bearing influence on rotor dynamics. They stressed the need for experimental work in the field of rotor dynamics to study the influence of bearings and supports upon the rotor response, in particular for full-scale rotor systems. Lund (1979, 1987) gave a review on the theoretical and experimental methods for the determination of the fluid-film bearing dynamic coefficients. For experimental determination of the coefficients, he suggested the necessity of accounting for the impedance of the rotor. Stone (1982) gave the state of the art in the measurement of the stiffness and damping of rolling element bearings. He concluded that the most important parameters influence the bearing coefficients were type of bearing, axial preload, clearance/interference, speed, lubricant and tilt (clamping) of the rotor. Kraus et al. (1987) compared different methods (both theoretical and experimental) to obtain axial and radial stiffness of rolling element bearings and showed a considerable amount of variation by using different methods. Someya (1976) compiled extensively both analytical as well as experimental results (static and dynamic parameters) for various fluid-film bearing geometries (e.g. 2-axial groove, 2-lobe, 4 & 5-pad tilting pad). Goodwin (1991) reviewed the experimental approaches to rotor support impedance measurement. He concluded that measurements made by multi-frequency test signals provide more reliable data. Swanson and Kirk (1997) presented a survey in tabular form of the experimental data available in the open literature for fixed geometry hydrodynamic journal bearings. Recently, Tiwari et al. (2004) gave a review of the identification procedures applied to bearing dynamic parameters estimation. The main emphasis was given to summarise various bearing models, the existing
experimental techniques for acquiring measurement data from the rotor-bearing test rigs, theoretical procedures to extract the relevant bearing dynamic parameters and to estimate associated parameters uncertainties. They concluded that the synchronous unbalance response, which can easily be obtained from the run-down/up of large turbomachineries, should be exploited more for the identification of bearing dynamic parameters along with the estimation of residual unbalance.

Until the early 1970s the usual method for to obtain the dynamic characteristics of systems was to use sinusoidal excitation. In 1971 Downham and Woods (1971) proposed a technique using a pendulum hammer to apply an impulsive force to a machine structure. Although they were interested in vibration monitoring rather than the determination of bearing coefficients, their work is of interest because impulse testing was thought to be capable of exciting all the modes of a linear system. Due to the wide application of the FFT algorithm and the introduction of the hardware and software signal processor, the testing of dynamic characteristics by means of transient excitation is now common. Morton (1975a and 1975b) developed an estimation procedure for transient excitation by applying step function forcing to the rotor. With the help of a calibrated impact of known breaking load, the sudden removal of the load on the rotor in the form of a step function (broad band excitation in the frequency domain) was used to excite the system. The Fourier transform was used to calculate the FRFs in the frequency domain. He assumed the bearing dynamic parameters to be independent of the frequency of excitation. The analytical FRFs, which depend on the bearing dynamic coefficients, were fitted to the measured FRFs. He also included the influence of shaft deformation and shaft internal damping into the estimation of dynamic coefficients of bearings. Chang and Zheng (1985) used a similar step-function transient excitation approach to identify the bearing coefficients and they used an exponential window to reduce the truncation error in the FFT due to a finite length forcing step-function. Zhang et al. (1988) used the impact method with a different fitting procedure to reduce the computation time and the uncertainty due to phase-measurement. They quantified the influence of measurement noise, the phase-measuring error and the instrumentation reading drift on the estimation of bearing dynamic coefficients. Marsh and Yantek (1997) devised an experimental set-up to identify the bearing stiffness by applying known excitation forces (e.g. measured impact hammer blows) and measuring the resulting responses by accelerometers. They estimated the bearing stiffness of rolling element bearings (consisting of four recirculating ball bearing elements) of a precision machine tool using the FRFs. The tests were conducted on a specially designed test fixture (for the non-rotating bearing case). They stressed experimental issues such as the precise location of the input and output measurements, sensor calibration, and the number of measurements. Among the experimental methods, the impact excitation method proposed by Nordmann and Scholhorn (1980) to identify stiffness and damping coefficients of journal bearings is the most economical and convenient. Impulse force has an advantage that is, it contains many excitation frequencies simultaneously and a single impact force can excite several modes. In this work analytical frequency response functions, which
depend on bearing dynamic coefficients are fitted to measured responses. Stiffness and damping coefficients are the results of an iterative fitting process. Burrows and Sahinkaya (1982) showed that the frequency domain bearing dynamic parameters identification techniques are less susceptible to noise. Zhang et al. (1990) and Chan and White (1990) used the impact method to identify bearing dynamic coefficients of two symmetric bearings by curve fitting frequency responses. Arumugam et al. (1995) extended the method of structural joint parameter identification method proposed by Wang and Liou (1991) identified the eight-linearised oil-film coefficients of tilting pad cylindrical journal bearings utilizing the experimental frequency response functions (FRFs) and theoretical FRFs obtained by finite element modelling. Qiu and Tieu (1997) used the impact excitation method to estimate bearing dynamic coefficients of rigid rotor system from impulse responses. 

Advances in the sensor technology and increase in the computing power in terms of the amount of data could be collected/handled and the speed at which it can be processed leads to the development of methods that could be able to estimate residual unbalance along with bearing/support dynamic parameters simultaneously (Chen and Lee, 1995; Lee and Shih, 1996; Tiwari and Vyas, 1997; Sinha et al., 2002). These methods could be able to estimate residual unbalances quite accurately but estimation of bearing dynamic parameters often suffers from scattering due to the ill-conditioning of the regression matrix of the estimation equation (Edwards et al., 2000 and Sinha et al., 2002).

**Concluding Remarks**

To summarise, in the present chapter a detailed treatment is given to methods of estimating dynamic parameters of bearings and seals. Simple methods of exciting the rotor by the static to dynamic forces are described for the rigid shaft case as well as for flexible shafts. Dynamic force methods are found to be more reliable. Vibration shakers, impact hammers, and unbalances are some of the ways by which the force can be given to the rotor-bearing system. Methods are described for all these ways of excitation either to the rotor or to the floating bearing. Dynamic forces of various kinds are considered, e.g., sinusoidal, bi-frequency, multiple frequency, impulse, and random. Multiple frequency tests are found to be suitable in terms of exciting several modes of the system simultaneously to have more representative vibration signal available for processing to get reliable bearing dynamic parameters. However, the unbalance response is shown to be more practical means of getting the vibration signals from the real machines. Sensitivity of the estimated parameters is considered.
Exercise Problems

Exercise 14.1 For the estimation of bearing stiffness coefficients by the static load method, the static load of 400 N is applied in the vertical and horizontal directions, one at a time. When the load is applied in the horizontal direction, it produces displacements of 22 μm and −20 μm in the vertical and horizontal directions respectively, whilst the vertical load produces respective displacements of 4 μm and 12 μm. Obtained bearing stiffness coefficients from the above measurements.

*Answer:* The stiffness coefficients are $k_{xx} = -4.651 \text{ MN/m}$, $k_{xy} = 13.953 \text{ MN/m}$, $k_{yx} = 25.581 \text{ MN/m}$ and $k_{yy} = 23.256 \text{ MN/m}$.

Exercise 14.2 A test rig is used to measure the hydrodynamic bearing stiffness coefficients by applying first of all a horizontal load of 400 N. It produces displacements of 10 μm and 4 μm in the horizontal and vertical directions, respectively. Then in second case only a vertical load of 300 N is applied. It produces displacements of -20 μm and 20 μm in the horizontal and vertical directions, respectively. Calculate the value of the stiffness coefficients based on these measurements.

*Answer:* The elements of complex receptance matrix for the bearing are: $R_{xx} = 4.5736 \times 10^{-8} - j9.2712 \times 10^{-9} \text{ m/N}$, $R_{xy} = 5.2734 \times 10^{-8} + j7.97 \times 10^{-9} \text{ m/N}$, $R_{yx} = 1.2656 \times 10^{-7} - j4.1942 \times 10^{-8} \text{ m/N}$ and $R_{yy} = 1.6559 \times 10^{-7} - j5.1224 \times 10^{-8} \text{ m/N}$

Exercise 14.3 A bearing is forced in the horizontal direction by a force $F_x = 200 \sin 150\pi t$ N. The resulting journal vibrations are $x = 12 \times 10^{-6} \sin(150\pi t - 0.35)$ m (in the horizontal direction) and $y = 20 \times 10^{-6} \sin(150\pi t - 0.4)$ m (in the vertical direction). When the same force is applied in the vertical direction the horizontal and vertical displacements take the respective forms $x = 13 \times 10^{-6} \sin(150\pi t + 0.3)$ and $y = 25 \times 10^{-6} \sin(150\pi t - 0.38)$. Determine elements of the complex impedance matrix for the bearing. *Answer:* The elements of complex impedance matrix for the bearing are $k_{xx} = 5061762.20178 + j27691680.0054 \text{ N/m}$, $k_{xy} = 7007752.44721 - j12851849.7486 \text{ N/m}$, $k_{yx} = -4491637.24935 - j22067930.6871 \text{ N/m}$, $k_{yy} = 2029852.21747 + j13358903.8911 \text{ N/m}$

Exercise 14.4. For the bearing dynamic parameter estimation, how many minimum numbers of independent sets of force-response measurements are required? Justify your answer. (Assume there is no residual unbalance in the rotor).

Exercise 14.5 The eight bearing stiffness and damping coefficients are to be determined by using the method described above. Experimental measurements of journal vibration amplitude and phase lag angle are given in the Table E14.5; pedestal vibrations are found to be negligible. Determine the
values of oil-film coefficients implied by these measurements, and the maximum change in the direct
cross-coupling terms introduced by an error of +4° in the measurement of the phase recorded as 42.5°.

<table>
<thead>
<tr>
<th></th>
<th>Forward excitation</th>
<th>Reverse excitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal vibration amplitude</td>
<td>66.4 µm</td>
<td>46.6 µm</td>
</tr>
<tr>
<td>Horizontal phase lag</td>
<td>42.5°</td>
<td>20.9°</td>
</tr>
<tr>
<td>Vertical vibration amplitude</td>
<td>55.5 µm</td>
<td>38.4 µm</td>
</tr>
<tr>
<td>Vertical phase lag</td>
<td>9.9°</td>
<td>111°</td>
</tr>
<tr>
<td>Force amplitude</td>
<td>1.0 KN</td>
<td>1.0 KN</td>
</tr>
<tr>
<td>Forcing frequency</td>
<td>12.6 Hz</td>
<td>12.6 Hz</td>
</tr>
<tr>
<td>Journal mass</td>
<td>150 kg</td>
<td>150 kg</td>
</tr>
</tbody>
</table>

MATLAB Solution:

**INPUT FILE**

```
% Name of this input file is input_qus_1_7.m
X1=66.4*1.0e-6; % horizontal vibration amplitude (in meter)
A1=42.5; % horizontal phase lag (in degree)
Y1=55.5*1.0e-6; % vertical vibration amplitude (in meter)
B1=9.9; % vertical phase lag (in degree)
F1=1*1.0e+3; % force amplitude (in N)
n1=12.6; % forcing frequency(in Hz)
M=150; % journal mass (in Kg)

% For the reverse excitation condition.
X2=46.6*1.0e-6; % horizontal vibration amplitude (in meter)
A2=20.9; % horizontal phase lag (in degree)
Y2=38.4*1.0e-6; % vertical vibration amplitude (in meter)
B2=111; % vertical phase lag (in degree)
F2=1*1.0e+3; % force amplitude (in N)
n2=12.6; % forcing frequency (in Hz)
M=150; % journal mass (in Kg)
```

**MAIN FILE**

```
clear all;
input_qus_1_7;
w1=2*pi*n1;
w2=2*pi*n2;
a1=A1/(pi/180);
b1=B1/(pi/180); % angular frequency
a2=A2/(pi/180);
b2=B2/(pi/180);

p=[-X1*sin(a1) Y1*cos(b1) 0 0 w1*X1*cos(a1) w1*Y1*sin(b1) 0 0; 0 0 -X1*sin(a1) Y1*cos(b1) 0 0 w1*X1*cos(a1) -w1*Y1*sin(b1) 0; 0 0 0 X1*cos(a1) Y1*sin(b1) 0 0 w1*X1*sin(a1) -w1*Y1*cos(b1) 0; -X2*sin(a2) Y2*cos(b2) 0 0 w2*X2*cos(a2) w2*Y2*sin(b2) 0 0; 0 0 0 0 -X2*sin(a2) Y2*cos(b2) 0 0 w2*X2*sin(a2) -w2*Y2*cos(b2) 0; 0 0 X2*cos(a2) Y2*sin(b2) 0 0 w2*X2*sin(a2) -w2*Y2*cos(b2) 0; 0 0 0 0 0 X2*cos(a2) Y2*sin(b2) 0 0 w2*X2*sin(a2) -w2*Y2*cos(b2)];
f=M* w1^2* X1^2*sin(a1); F1+M* w1^2* Y1*cos(b1); F1+M* w1^2* X1 *cos(a1); M* w1^2* Y1*sin(b1); -M* w2^2* X2^2*sin(a2); F2+M* w2^2* Y2*cos(b2); F2+M* w2^2* X2 *cos(a2); M* w2^2* Y2*sin(b2); k=pf;
disp ('The bearing coefficients are');
fprintf ('\nK_{xx}=%d N/m\n'), k
```

Table E14.5 Some test data used to calculate bearing stiffness and damping coefficients
The bearing stiffness and dynamic coefficients are:

\[
\begin{align*}
K_{xx} &= 32899302.6948 \text{ N/m} \\
K_{xy} &= 14491075.2905 \text{ N/m} \\
K_{yx} &= -9134864.0493 \text{ N/m} \\
K_{yy} &= -19410190.4825 \text{ N/m} \\
C_{xx} &= 115385.6165 \text{ N/m} \\
C_{xy} &= 257052.0267 \text{ N/m} \\
C_{yx} &= 403687.2122 \text{ N/m} \\
C_{yy} &= 183041.7669 \text{ N/m}
\end{align*}
\]

**Exercise 14.6** Bearing parameter estimation experimentation took place in two steps (i) the static method to determine the stiffness coefficients: for a force of \(f_x = 100 \text{ N}\) when applied to the rotor in horizontal direction it produced displacements of the shaft relative to the bearing \(u_x = 30 \text{ m}\) and \(u_y = 10 \text{ m}\), and for a force of \(f_y = 150 \text{ N}\) when applied to the rotor in vertical direction it produced displacements of the shaft relative to the bearing \(u_x = -20 \text{ m}\) and \(u_y = 40 \text{ m}\). (ii) the unbalance force method to determine the damping coefficients: for an unbalance of 6 gm-cm at 30° it produced displacements of the shaft relative to the bearing \(u_x = 50 \text{ m}\) with the phase of 140° and \(u_y = 30 \text{ m}\) with the phase of 330°.

Exercise 14.6 Choose a single answer from multiple choice answers

(i) For a speed-dependent bearing, the dynamic parameter estimation by using unbalance responses could be obtained by

(A) taking measurements at rotor speeds well separated with each other
(B) taking measurements at rotor speeds quite close to each other
(C) taking measurements with difference unbalance level at a constant rotor speed
(D) Both (B) and (C)

(ii) For bearing parameter estimation by using the static method, we can get

(A) Both the stiffness and damping coefficients
(B) Only stiffness coefficients
(C) Only damping coefficients
(D) none of them

(iii) For bearing parameter estimation by using the dynamic method, we can get

(A) Both the stiffness and damping coefficients
(B) Only stiffness coefficients
(C) Only damping coefficients
(D) none of them

(iv) An impulse force contains

(A) No frequency
(B) A single frequency
(C) two frequency
(D) multi-frequency
REFERENCES


Hummel, C., 1926, Kristische Drehzahlen als Folge der Nachgiebigkeit des Schmiermittels im Lager. VDI-Forschungsheft, 287.


Stodola, A., 1925, Kritische Wellenstörung infolge der Nachgiebigkeit des Ölpolslers im Lager (Critical shaft perturbations as a result of the elasticity of the oil cushion in the bearings). *Schweizerische Bauzeitung* **85**, No. 21, May.


