In Chapters 15 and 16, the measurement and signal processing techniques, transducers, signal conditioners and signal analysis equipments are described, which are used in the rotating machinery condition monitoring. It is very important to display the measured signal in convenient form so as to be useful for the interpretation of the condition of rotating machinery. In the present chapter, by looking at various forms of measured signal possible condition of machinery is provided. It also looks into correlation of a particular signature with a particular failure in more details regarding, which helps in assigning a particular failure. Every faults have a specific signature in the measured signal and it is most convenient and cheapest way to identify possible fault in a machinery. Now very advanced signal processing techniques (Wavelet transform, Genetic algorithms, Neural network, fuzzy logic and machine support vectors) are being applied in laboratory test set ups to detect, locate and quantify the faults and based on this even the life of the machinery is also being predicted. A brief review to application these techniques for rotating machinery condition monitoring is provided since detailed treatment to these newly emerging methods is beyond the scope of the present book. Proactive action to prevent a failure is the better the detection of failure. The next chapter will be dealt with introduction of the active control of rotors by magnetic bearings, which is still a research and applied area in the field of rotor dynamics.

Many rotating machines, such as power station turbogenerators, may be considered as consisting of three major parts; the rotor, the bearings (often fluid bearings) and the foundations. In many modern systems, the foundation structures are flexible and have a substantial influence on the dynamic behaviour of the complete machine. These rotating machines have a high capital cost and hence the development of condition monitoring techniques is very important. Vibration based identification of faults, such as rotor unbalance, rotor bends, cracks, rubs, misalignment, fluid induced instability, based on the qualitative understanding of measured data, is well developed and widely used in practice. However the quantitative part, the estimation of the extent of faults and their locations, has been an active area of research for many years. Over the past three to four decades, theoretical models have played an increasing role in the rapid resolution of problems in rotating machinery.

methods for the detection of defects in rolling element bearings was presented by Tandon and Choudhury (1999). The application of wavelets has emerged in the context of damage detection, and an excellent review of this is given in (Staszewski, 1998). A review of the research work performed in real-time active balancing and active vibration control for rotating machinery, as well as the research work on dynamic modeling and analysis techniques of rotor systems, is presented by Zhou and Shi (2001).

### 17.1 Rotor unbalance

In any rotating machinery the rotor unbalance is always present and it is one of the most common sources of severe vibration. The unbalance is defined in chapter 2, which is the product of the rotor mass and its eccentricity (the eccentricity is a distance of the centre of gravity of the rotor from its centre of rotation). When a severely unbalanced rotor is rotated freely on frictionless bearings, it stops at nearly fixed orientation. It indicates that the unbalance force acting at centre of gravity pulls the rotor to a fixed orientation due to its eccentricity. The position of the unbalance is also called the heavy (or hot) spot. The point on an unbalanced rotating shaft with the maximum displacement to the center of rotation (Figure 17.1) is called the high spot. It is observed by a vibration pickup as the point of maximum positive amplitude. The high and heavy spot may or may not coincide, depending on where the rotor is operating relative to its critical speeds. In the Jeffcott rotor it is observed that the high and heavy spot coincide below the critical speed and they are opposite above the critical speed. At critical speed they have 90° phase with heavy spot leading.

**Example 17.1** Consider a two-DOF Jeffcott rotor mounted on two identical flexible bearings. The mass of disc is 54.432 kg and the stiffness of the shaft is 1.378×10⁷ N/m. Consider the following properties of the each bearing: \( k_{xx} = 1.01×10^7 \) N/m, \( k_{yy} = 4.16×10^7 \) N/m, \( k_{xy} = 4.16×10^5 \) N/m, and \( k_{yx} = 3.12×10^7 \) N/m. Obtain the amplitude and phase variation with respect to the spin speed of the shaft. Plot the orbit (x-y plot) of the shaft by indicating its direction of rotation between critical speeds. Choose a suitable unbalance on the disc to generate the responses.

**Solution:** The unbalance response is generated by the procedure described in Chapter 4 and are shown in Figs. 17.1 and 17.2. It can be observed that the rotor-bearing system has two critical speeds (one around 400 rpm and other around 900 rpm). It can be seen that there are amplitude peaks corresponding to these critical speeds accompanied by phase changes. Figs. 17.3(a-c) show orbit plots for various spin speed of the rotor. It can be seen that there is a change in the sense of orbit rotation while crossing the critical speeds.
Fig. 17.1 (a) Variation of amplitude with spin speed of the shaft

Fig. 17.1 (b) Variation of amplitude with spin speed of the shaft
Fig. 17.2 (a) Variation of amplitude with spin speed of the shaft

Fig. 17.2 (b) Variation of amplitude with spin speed of the shaft
Fig. 17.3(a) Orbit plot for rotor spin speed of 200 rpm (below the first critical speed)

Fig. 17.3(b) Orbit plot for rotor speed of 400 rpm (between the first and second critical speed)

Fig. 17.3(b) Orbit plot for rotor speed of 1000 rpm (above the second critical speed)
The vibration occurs at machine rotational frequency, in general, but sometimes higher-harmonics of rotational speed are excited. Machine vibration caused by unbalance can mostly be detected by monitoring shaft displacement amplitude and phase as the machine is run through its critical speed, filtering out non-rotational speed frequencies by using tracking filter. The amplitude peaks at the critical speed, and phase changes by $180^\circ$ passing through about $90^\circ$ at the critical and it can be seen in the Bode plot (see Figure 17.1-17.2). The shaft whirl orbit takes (generally) on an ellipse shape (unless the shaft support impedance is isotropic, in which case it is circular), the orientation of the ellipse changes as the critical speed is passed through (see Figure 17.3). Already balancing of rotors (a systematic approach for quantifying the unbalance) has been covered in great details in Chapter 13, which must be performed after every major overhauling of the machinery. A review of the research work performed in real-time active balancing and active vibration control for rotating machinery, as well as the research work on dynamic modeling and analysis techniques of rotor systems, is presented by Zhou and Shi (2001). The basic methodology and a brief assessment of major difficulties and future research needs are also provided.

In some instances the magnitude and phase of the unbalance vibration vector might change with time. When symptoms of unbalance exhibit this feature the correct treatment is not normally to simply re-balance the machine. Example of fault in some categories are (i) the heavy spot on the shaft causing a thermal bow, it occurs when the heavy rotors of turbine are left non-rotated for cooling during shutdown and due to heavy sag it get permanent bent configuration or even when the rotor has axial asymmetry of the thermal distribution (ii) components mounted loosely on the shaft e.g. discs, gears, flywheel, etc.; and (iii) moisture entering a hollow shaft, which vary in amount and location during the operation of the machinery.

17.2 Shaft Bow or Thermal Bow
The phenomenon of shaft forcing due to an initial bend has aroused interest over the last three decades years or so, albeit much less than mass unbalance. Bends in shafts may be caused in several ways, for example due to creep, thermal distortion or a previous large unbalance force. The forcing caused by a bend is similar, though slightly different, to that caused by conventional mass unbalance. There have been numerous cases in industry where vibration has been assumed to have arisen from mass unbalance and rotors have been balanced using traditional balancing procedures. This has repeatedly left engineers puzzled as to why vibration persists after balancing, and vibration levels may indeed even be worse than before balancing took place. Bend response is independent of shaft speed and causes different amplitude and phase angle relationships than is found with ordinary mass unbalance, where the forcing is proportional to the square of the speed Nicholas et al. (1976). In the case of a bent rotor, the excitation is proportional to the magnitude of the bow along the rotor. A bent rotor gives rise to synchronous excitation, as with mass unbalance, and the relative phase between the bend
and the unbalance causes different changes of phase angle through resonance than would be seen in the pure unbalance case, as described in references Nicholas et al. (1976). It is therefore important to be able to diagnose a bend in a rotor from vibration measurements and thus distinguish between it and mass unbalance. The identification method was extended to include the identification of a bend in a rotor, as well as estimating the unbalance and flexible support parameters of the rotor-bearing-foundation system by Edwards et al. (2000). They expressed the rotor bend in terms of the free-free eigenvectors of the rotor, which are readily obtained from the numerical model. It is considered that, even for relatively simple systems with low numbers of modes, these modes will still be sufficient to adequately represent complicated bend geometry. The force due to the bend is then expressed in terms of shaft stiffness and bend geometry, which contain the unknown modal coefficients to be estimated during the process of identification.

One of the typical reasons for an unbalance in a rotor is due to warping or static bending (permanent or bending). Such rotors are called warped or bowed rotors and are encountered in practice for different reasons, e.g., allowing a horizontal rotor rest for long period time or due to rubbing of a shaft on a seal. Though the effect of a bent is similar to unbalance conditions, it is different from the classical eccentricity of the mass from the geometric centre. The bow and the eccentricity are in general in different angular locations and the warped rotor behave somewhat differently from purely unbalance rotors. In important investigation of such rotor is performed by Nicholas et al. (1976), Ehrich (1992) and Rao (2001) both analysis and diagnostics point of view.

The effects of most typical faults of rotating machines can be simulated by means of suitable sets of equivalent excitations, forces or moments, that are applied to nodes of the finite element model of the shaft-train. When a beam finite elements are used, each node of the beam elements has four-DOF: two orthogonal radial displacements (lateral vibrations) and rotations (associated with a shaft bending in two orthogonal planes). The equivalent excitations are referred to an absolute reference system and are denoted $\mathbf{F}_n(\omega)$. This vector can be written as (Pennacchi and Vania, 2004)

$$
\mathbf{F}_n(\omega) = \mathbf{F}(n\omega) = \mathbf{F}_n(\omega)e^{i\omega t}
$$

(11.1)

Depending on the type of fault the equivalent excitations can be expressed by a rotating vector, with a certain harmonic content, or by a fixed vector. In general, in the absence of nonlinear effects, the frequency spectrum of the equivalent excitations contains significant harmonic components limited to the rotating frequency (1x) and to integer multiples of this frequency (i.e. 2x, 3x).
A bowed rotor or a permanent bent rotor can be considered as the Jeffcott rotor model with a static deflection from the bearing centre line with a magnitude of $e_0$ at a phase angle $\phi_0$ with the unbalance as the reference line. This gives the equation of motion of the following form (Rao, 2001)

$$m\ddot{s} + c\dot{s} + ks = me\omega^2 e^{i\omega t} + ke_0 e^{i(\omega t + \phi_0)}$$  \hspace{1cm} (11.2)

where the whirl radius is defined as $s = x + jy$, $x$ and $y$ are displacements of the disc mass in the horizontal and vertical directions, respectively. Force components consist of two parts, the first due to the rotor unbalance ($me\omega^2 e^{i\omega t}$) and the second due to rotor bow $\{ke_0 e^{i\omega t}\}$. On dividing by $me$, equation (11.2) can be written as

$$\ddot{s} + 2\zeta\omega_0 \dot{s} + \omega_0^2 s = (\omega^2 + \omega_0^2 e^{i\phi_0}) e^{i\omega t}$$  \hspace{1cm} (11.3)

with

$$s = s/e, \quad \omega^2 = k/m, \quad \omega_0 = e_0/e, \quad \zeta = c/e, \quad c_e = 2\sqrt{km}$$  \hspace{1cm} (11.4)

The solution can be written as

$$s = \frac{s}{e} = \frac{\omega^2 + \omega_0^2 e^{i\phi_0}}{\sqrt{(1 - \omega^2)^2 + (2\zeta\omega)^2}} e^{i(\omega t - \phi)}$$  \hspace{1cm} (11.5)

It should be noted that the phase reference is the unbalance location. Hence, the amplitude of the unbalance response and the bow response amplitudes, respectively, are

$$s_{unb} = \frac{\omega^2}{\sqrt{(1 - \omega^2)^2 + (2\zeta\omega)^2}} \quad \text{and} \quad s_{bow} = \frac{\omega_0}{\sqrt{(1 - \omega^2)^2 + (2\zeta\omega)^2}}$$  \hspace{1cm} (11.6)

and phases, respectively, are

$$\phi_{unb} = 2\zeta\omega \quad \frac{1 - \omega^2}{1 - \omega_0^2} \quad \text{and} \quad \phi_{bow} = \phi - \phi_0$$  \hspace{1cm} (11.7)

Hence, the effect of bow would be to give the same frequency as that of the spin speed of the shaft, however, it may change the unbalance response and its phase depending upon the phase of the bow.
17.3 Misalignment

Unbalance and misalignment (the objective of the alignment is to have two coupled shafts perfectly collinear under operating conditions or between the bearing and the shaft their axis should be collinear or two bearings carrying a common shaft should have their axis collinear) are the two most common sources of machinery vibration. Accordingly, misalignments can be classified as (i) parallel (ii) angular, and (iii) combination of parallel and angular misalignments (Figs. 17.4 and 17.5). Like the unbalance, the misalignment is an installation and subsequent maintenance problem, since it can be corrected and prevented by using the proper installation and maintenance procedures. Shafts with a heavy pre-load carried by the bearings (e.g., in angular contact ball bearings in tandem the preloading must be applied to keep the bearing in assembled position), as distinct from the out-of-balance load, can show variation characteristics similar to those caused by bearing misalignment (Figure 17.6). This category of fault is probably the second most common cause of machine vibration, after the unbalance. A pre-load might also be applied at a bearing as a consequence of gear-mesh forces, aerodynamic forces and hydrodynamic forces. Misalignment may be present because of improper machine assembly or as a consequence of thermal distortion, and it results in additional loads being applied to the bearing.

![Parallel Misalignment](image1)

**Parallel Misalignment**

![Angular Misalignment](image2)

**Angular Misalignment**

![Combined Misalignment](image3)

**Combined Misalignment**

Fig. 2. Schematic of rotor with misalignment at a coupling

Fig 17.4 (Sinha et al., 2004)
The general perception and observation is that misalignment in multi-coupled rotors generates a $2\times$ (twice the rotating speed) component in the response of the machine (Dewell and Mitchell, 1984; Ehrich, 1992) and the effect on the $1\times$ component is assumed to be small. However, Jordan (1993) confirms that the misalignment initially affects the $1\times$ response resulting in an elliptical orbit, but in the case of severe misalignment the orbit plot may look like a figure eight due to the appearance of a $2\times$ component in the response. These features are usually used for the detection of the presence of rotor misalignment (Jordan, 1993). In practice both the $1\times$ and the $2\times$ response will be affected by misalignment, and the physical source of these effects may be modelled as a rotor bend and rotor asymmetry, respectively. In this paper the identification of rotor unbalance and misalignment uses the $1\times$ response at the bearing pedestals of the machine from a single rundown, even though the direct influence on the $1\times$ response due to misalignment may be small.

Misalignment of adjoining shafts, or of the bearings of one shafts, causes abnormal loads to be transmitted through the bearings, and imposes additional bending stress on the shaft thereby reducing its fatigue life. In some cases misalignment may cause one bearing to be unloaded (if the pre-load so applied is in the opposite direction to the normal gravity loads, this can result in lowering the machine critical speed or even instability in the system. These symptoms are sometimes present in addition to that of excessive vibration. The vibration associated with misalignment occurs at $1\times$ machine running speed but, unlike unbalance case, there is usually a substantial component in the axial direction, which
may be greater than radial direction. Substantial amounts of misalignment (or pre-load) can also cause vibrations at frequencies of $2 \times$ machine running speed, and sometimes higher multiplies. When the amplitude at twice the machine running speed exceeds 150 percent of the amplitude at running frequency, the misalignment is producing a severe action at coupling.

![Figure 17.6 Cascade plot with shaft misalignment](image)

Vibrations caused by misalignment tend to peak near the machine critical speed, but away from the critical may either stay about constant or tend to increase, in many cases they have been reported to appear suddenly as machine speed is increased, and to disappear equally quickly for a particular speed range. For moderate misalignment the shaft amplitude is reduced in the sense of the applied pre-load, so that if it were originally circular it might become elliptical; that is because there is more resistance to motion in the direction opposing the preload so the rotor ‘feels’ a higher support stiffness in this direction. For more severe cases the orbit may become banana-shaped or Figure-8 shaped (Figure 17.7). If the misalignment has been introduced when two shafts were improperly coupled, some reduction of the undesirable symptoms may be obtained by using a flexible coupling; the amount of improvement obtained will depend upon the coupling stiffness. Bearing temperature and oil-film pressure are other indirect methods of detecting misalignment. Coupling alignment may be checked by taking dial gauge readings and gap measurements (Piotrowski, 1986). The variations in these measurements should be recorded as both shafts rotated simultaneously, and averaged values used to
calculate the misalignment present. Alignment of bearings may be checked using optical/laser methods (one bearing with another) or using feeler gauges in the clearance between journal and bush. Also proximity transducer measurement may be used to record the gap between the shaft and each end of a bearing and so indicate alignment. Other indirect indicators of alignment in fluid bearings are measurements of the bearing temperature and lubricant pressure distribution.

Misalignment of machinery shafts causes reaction forces to be generated in the coupling which affect the machines and are often a major cause of machinery vibration. Bloch (1976) identified the forces and moments developed by a misaligned gear coupling. Gibbons (1976) showed that these forces and moments are developed by different types of misaligned couplings. Comparative values of these forces were presented. The effect that these forces upon the machines has been described in general terms. Schwerdlin (1979) found that the manufacturers published ratings for flexible couplings typically do not take into account reaction forces from misalignment, speed, and torque. In general, these forces must be determined by tests in order to reveal their influence on bearings, shafts and other components in the drive. Shekhar and Prabhu (1995) modeled the rotor-bearing system using higher-order finite elements by considering deflection, slope, shear force, bending moment with eight DOFs per node. The reaction forces, moments developed due to flexible coupling misalignment were derived and introduced in the model. The unbalance response in two harmonics was evaluated. The increase in harmonics with misalignment was modeled by using FEM analysis. The location of the coupling with respect to the bending mode shape had a strong influence on the vibrations. The bending 1× vibration response showed that coupling misalignment did not significantly alter the amplitude. The 2× vibration response showed the characteristic signature of misaligned shafts. The theoretical model for the coupling-rotor-ball bearing systems with misalignment was derived by Lee and Lee (1999), including the loads and deformations of the bearings as well as the flexible coupling as the misalignment effects. Throughout the extensive experimental and simulation works, the validity of the model was validated and the rotor dynamic characteristics related to misalignment were investigated. Prabhiakar et al. (2002) used the finite element method (FEM) for transient analysis with crack and coupling misalignment separately for the same rotor to distinguish crack from coupling misalignment. Continuous wavelet transform (CWT) was used to extract characteristic features from vibration response of these two flaws in rotor system. A method to estimate both the rotor unbalance (amplitude and phase) and the misalignment of a rotor–bearing–foundation system was presented by Sinha et al. (2004). The estimation uses a priori rotor and bearing models along with measured vibration data at the bearing pedestals from a single rundown or run-up of the machine. The method also estimates the frequency-band-dependent foundation parameters to account for the dynamics of the foundation. The suggested method has been applied to a small experimental rig and the estimated results were validated. It assumed that the rotor misalignment occurs at the couplings between the
multi-rotors. The nature of the rotor misalignment could be parallel, angular or combined as shown in Fig. 17.4, but all of them would generate forces and/or moments. The method estimated the forces and moments due to the rotor misalignment. However, they considered displacements and angles rather than forces and moments, although this required the stiffness matrix of the coupling. Suppose that the stiffness of the $i^{th}$ coupling is $[K_{i,j}]$; then the linear misalignment, $\Delta x_i$ and $\Delta y_i$; and the angular misalignment, $\Delta \varphi_{x,i}$ and $\Delta \varphi_{y,i}$; at the $i^{th}$ coupling in the horizontal and vertical directions may be calculated as

$$\begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta \varphi_{x,i} \\ \Delta \varphi_{y,i} \end{bmatrix} = [K_{i,j}]^{-1} \begin{bmatrix} f_{x,i} \\ f_{y,i} \\ M_{x,i} \\ M_{y,i} \end{bmatrix}$$ (11.8)

Pennacchi and Vania (2005) focused on the application of two different diagnostic techniques aimed to identify the most important faults in rotating machinery as well as on the simulation and prediction of the frequency response of rotating machines. The application of the two diagnostics techniques, the orbit shape analysis and the model based identification in the frequency domain, was described by means of an experimental case study that concerns a gas turbine-generator unit of a small power plant whose rotor-train was affected by an angular misalignment in a flexible coupling, caused by a wrong machine assembling. The fault type was identified by means of the orbit shape analysis, then the equivalent bending moments, which enabled the shaft experimental vibrations to be simulated, have been identified using a model based identification method. These excitations was used to predict the machine vibrations in a large rotating speed range inside which no monitoring data were available. The results obtained emphasise the usefulness of integrating common condition monitoring techniques with diagnostic strategies.

17.4 Rubs

Rubs are produced when the rotating shaft comes into contact with the stationary components of the machine. Rubs are said to be accompanied by a great deal of high-frequency spectral activities. A rub is generally transitory phenomenon. Rub may typically be caused by mass unbalance, turbine or compressor blade failure, defective bearings and/or seals, or by rotor misalignment, either thermal or mechanical. Several different physical events may occur during a period of contact between rotor and stator: initial impacting stage, frictional behaviour between the two contacting parts and an increase in the stiffness of the rotating system whilst contact is maintained, to name just three. The behaviour of the system during this period is highly non-linear and may be chaotic.
The rotor-to-stator rub is one of the malfunctions occurring often in rotating machinery. It is usually a secondary phenomenon resulting from other faults. When the rub occurs, partial rub can be observed at first. During one complete period, rotor and stator have rub and impact interaction once or a few times. Alternately changed stress is formed in the shaft and the system can exhibit complicated vibration phenomena. Chaotic vibration can be found under some circumstances. Gradual aggravation of the partial rub will lead to full rub and severe vibration makes the normal operation of the machine impossible. Mathematically, the rotor system with rotor-to-stator rub is a nonlinear vibrating system with piecewise linear stiffness. There have been many publications on this problem and relevant topics.

Rubs may be classified as partial rubs or full rubs or may take the form of a stick-slip action. A partial rub is that where contact between the shaft and stationary component exists only for part of the cycle time, for example the high spot on a shaft rubs against part of a seal, or a bowed rotor rubs the stator. A flattened waveform and orbit are strong indications of a partial rub. The frequency spectra for a partial rub always show some synchronous vibration (1× shaft rotational shaft) together with some sub-harmonics which are related to free lateral vibrations of the rotor. The sub-harmonics may be in either the forward or backward directions.

In case of a full rub, the shaft may no longer make contact with the stationary member only at local high spots, but instead bounce its way all around its orbit, for example in case of severe misalignment or due to severe unbalance which produces large unbalance response in the rotor. The main component is at 1× rotational speed; harmonics and sub-harmonics may also be present. Alternatively, the tangential friction force between shaft and stationary component may be so large that it results in a backward precession of the shaft around the inside of the stationary component together with substantial slipping.

Figure 17.8 An equivalent stick-slip rub in rotor-bearing system
A rub may also give rise to stick-slip action of the shaft against the stationary components (similar to the Coulomb friction). It is a vibration set-up as a consequence of the friction coefficient between shaft and stationary component changing with relative velocity. In the Figure 17.6 the rotor rotation initially causes it to roll over the stationary component so that its center moves from left to right. When the spring force in the shaft is equal to the friction force that initiates the motion then the rotor starts to slip over the stationary component instead of rolling over it. At this moment there is change in friction coefficient which leads to a smaller friction force and the rotor now moves from right to left, overshoots the point where the new friction force is equal the shaft spring force because of the inertia of the rotor. At some point the rotor starts moving from right to left and instead begins to roll on the stationary component once more, and the cycle of events starts again. The vibrational frequency is usually much higher than the shaft rotational frequency (Fig. 17.9), and the fault frequently shows up as a torsional vibration as well as a lateral one. In case where rubs cannot be avoided by increasing clearance then either the rotor surface or that of stationary component should be manufactured from a soft material which will easily wear away, this will help to avoid the backward precession. The situation may be avoided by lubricating the area of contact. Also by using very stiff rotor and flexible mounted stator above situation can be avoided.
Rotor-to-stationary element rub is a serious malfunction in rotating machinery that may lead to a catastrophic failure. Choy and Padovan (1987) developed a general analytical rub model using the following assumptions: (i) simple Jeffcott rotor model, (ii) linear stiffness and damping characteristics, (iii) rigid casing supported by springs acting in the radial direction, (iv) mass inertia of the casing small enough to be neglected, (v) simple Coulomb friction and (vi) onset of rub caused by imbalance. Using the above assumptions, the rubbing period was categorized into four distinct processes: (a) non-contact stage, (b) rub initiation, (c) rub interaction and (d) separation. The effect of including torsion in the numerical analysis of a contacting rotor/stator system was investigated by (Edwards, 1999). Two sets of data were obtained for the same physical model, with and without the inclusion of torsion. System response with respect to torsional stiffness has also been examined. The results presented showed that, for a realistic physical system, periodic non-linear and also chaotic motion can occur.

Rub-impact between rotor and stator is one of the main serious malfunctions that often occur in rotating machinery. It shows a very complicated vibration phenomenon, including not only periodic (synchronous and non-synchronous) components but also quasi-periodic and chaotic motions. A comprehensive investigation on the dynamic characteristics exhibited by this kind of system forms the basis to diagnose accurately this form of fault.

Rotor-stator rub in a rotating assembly has attracted great concerns from researchers. There have been numerous publications on this topic. Muszynska (1989) literature survey gave a list of previous papers on the rub-related vibration phenomena. Dynamic phenomena occurring during rubbing such as friction, impacting, stiffening and coupling effects were discussed in the paper. She discussed the physical meaning and the thermal effect of rub, various phenomena during rubbing, analysis and vibration response of rubbing rotors, and other related phenomena. Thermal effects, friction, impact, coupling, stiffening, analysis and vibrational response are included; the paper represents the first comprehensive review of the subject. Beatty (1985) suggested a mathematical model for rubbing forces which were non-linear with a piecewise linear form. The model is still widely used today. Through theoretical simulation and laboratory verification he concluded some points for diagnosing this fault. Choy and Padovan (1987) performed a very interesting theoretical investigation to observe the effects of casing stiffness, friction coefficient, unbalance load and system damping on rub force history, and the transient response of rotor orbit. Shaw and Holmes (1883) discussed a periodically forced piecewise linear oscillator in a more mathematical way. Their results showed harmonic, sub-harmonic and chaotic motions. This type of oscillator can be derived from a rub-impact model and the discussion has a representative meaning. Thompson and Stewart (1986) studied an impact oscillator. The oscillator is shown to exhibit complex dynamic behaviour including period-doubling bifurcation and chaotic motions. Choi and Noah (1994) examined the complex dynamic behaviour of a simple
horizontal Jeffcott rotor with bearing clearances. Numerical results revealed that alternating periodic, aperiodic, and chaotic responses is governed by the rule of the Farey number tree. There are mode-locking tongues in the parameter space and within each mode-locking tongue, a number of smaller tongues exist where a sequence of period doubling bifurcations leading to chaos takes place (Kim and Noah, 1990) used a modified Jeffcott model to determine the onset of aperiodic whirling motion using bifurcation theory. Choi and Noah (1987) presented a numerical method which combined the harmonic balance method with discrete Fourier transformation and inverse discrete Fourier transformation. Their numerical results showed the occurrence of super- and subharmonics in a rotor model involving a bearing clearance. Chancellor et al. (1998) discussed a method of detecting parameter changes of a piecewise-linear oscillator using analytical and experimental non-linear dynamics and chaos. Chaotic time series for each of six parameter values was obtained. Movement of the unstable periodic orbits in phase space was used to detect parameter changes in the system. This suggested a possible way for the future fault diagnostics. Muszynska (1984) analyzed the physical phenomena related to partial lateral rotor to stator rub. Through using a periodic step function the analysis showed the existence of harmonic vibrations in the order 1/2, 1/3, 1/4, …; experiment also confirmed the results. Adams and Abu-Mahfouz (1994) discussed the chaotic motions of a general rub-impact rotor model. The influence of clearance variation was observed, and responses rich in subharmonic, quasi-periodic and chaotic motions were obtained over a wide range of operating parameters. Ehrich (1994) analyzed the rotor dynamic response in non-linear anisotropic mounting systems which represented a rotor system in local contact with the stator. He found the chaotic response in transition zones between successive super-harmonic orders. In Isaksson’s paper (1994), the “jump” phenomenon and the influence of radial clearance were investigated. Chu and Zhang (1997) performed a numerical investigation to observe periodic, quasi-periodic and chaotic motions in a rub-impact rotor system supported on oil film bearings. Routes to and out of chaos were analyzed.

Based on these researches it is not so difficult to judge whether a rotor system has rubbing or not. For partial rub the vibration waveform of the rotor will have truncation. When the rub is developed into full rub, the rotor vibration shows backward orbiting, which is a special feature to identify the rotor-to-stator rub, distinguishing this malfunction from the others. However, for the diagnostics purpose, it is still a difficult task to detect the rubbing position. Wang and Chu (2001) presented an experimental method to detect the rubbing location. The method combined the acoustic emission technique with the wavelet transform. The results showed the method to be very effective. Chu and Lu (2002) discussed a so-called dynamic stiffness method to detect the rubbing location. The simulation results showed the method to be very effective as well. Peng et al. (2003) analyzed the feature extraction of the rub-impact system for the fault diagnostics by means of the wavelet analysis. Similar to this kind of nonlinear system with piecewise linear feature, Hinrichs et al. (1999) investigated an impact oscillator and a self-sustained friction oscillator by experiments and numerical simulation. Point-mapping
approaches, Lyapunov exponents and phase space reconstruction were used to analyze dynamics of the systems and rich bifurcational behavior was found. Bapat (1998) discussed N impact periodic motions of a single-degree-of-freedom oscillator. For some simple cases, exact closed-form expressions were obtained. Effects of amplitude and frequency of sinusoidal force, bias force, damping, and variable and constant coefficients of restitution on periodic motions were investigated. Begley and Virgin (1998) investigated the interaction and influence of impact and friction on the dynamic behavior of a mechanical oscillatory system. Dynamical system theory was used as a conceptual framework and comparisons were made between numerical and experimental results over a relatively wide range of parameters. An experimental setup was installed to simulate the rotor-to-stator rub of the rotor system by Chu and Lu (2005). A special structure of stator is designed that can simulate the condition of the full rub. The vibration waveforms, spectra, orbits and Poincaré’s maps are used to analyze nonlinear responses and bifurcation characteristics of the system when the rub-impact occurs. Experiments with different conditions, including one and two rotor with single- and multi-disks, are performed. Very rich forms of periodic and chaotic vibrations were observed. The experiments show that the system motion generally contains the multiple harmonic components such as 2X, 3X, etc. and the 1/2 fractional harmonic components such as 1/2X, 3/2X, etc. Under some special conditions, the 1/3 fractional harmonic components such as 1/3X, 2/3X, etc. were observed.

In practice, the rubbing can be detected indirectly by measuring regularly the rundown time of the turbines. Due to rubbing rundown time is expected to reduce drastically.

17.5 Mechanical Looseness of Components

Mechanical looseness, the improper between component parts, is generally characterized by a long string of harmonics of running frequency with abnormally high amplitudes. In some machines vibration levels may be excessive as a consequence of components being assembled too loosely, for example in the case of a bearing, which is not properly secured. The vibration signal detected from a transducer is normally a sine wave, but is truncated at one extremity as shown in Figure 17.10(a). The truncation of the sine wave occurs because of the non-linearity of, for example, the bearing mount that suddenly becomes very stiff when the loose component reaches the limit of its allowable levels. In frequency domain these truncation (impulses) show up as a series of harmonics of rotational frequency (Figure 17.10(b)). The 2× machine running speed component is usually the harmonic most easily detected. In cases where it is the bearing, which is not properly secured, the shaft average support stiffness over one cycle is effectively reduced. When the shaft speed is less than 2× normal resonant frequency, the reduction in shaft support stiffness acts to lower the resonant frequency to ½× shaft rotational speed so that vibration at this frequency becomes significant. A similar effect may occur with other sub-harmonics of rotational speed. (i.e., ¼, ⅛ etc.).
Pedestal looseness is one of the common faults that occur in rotating machinery. It is usually caused by the poor quality of installation or long period of vibration of the machine. Under the action of the imbalance force, the rotor system with pedestal looseness will have a periodic beating. This will generally lead to a change in stiffness of the system and the impact effect. Therefore, the system will often show very complicated vibration phenomena. In the case of rotor/bearing/stator systems, the dynamic phenomena (which include chaotic and orderly periodic motion of system elements), occur usually as secondary effects of a primary cause. This primary cause is most often an action of the rotor unbalance related rotating force, directly exciting rotor lateral vibrations, and/or the unilateral, radial force applied to the rotor. The latter may result from rotor misalignment, fluid environment side load (as in single volute pumps), or gravity (as in horizontal rotors). For example, the looseness in a machine pedestal joint, or in an oversize bearing, would not be noticed until the rotating unbalance force, which periodically acts vertically up, exceeded the gravity force that pressed the joints into close contact. In the rotor-to-stator rub case, the unwelcome rotor-to-stator contact occurs when the rotor is moved to the side due to increased radial force, or when amplitudes of its lateral, precessional motion exceeded allowable clearances.

There have been very few publications on this topic. Goldman and Muszynska (1991) performed experimental, analytical and numerical investigations on the unbalance response of a rotating machine with one loose pedestal. The model was simplified as a vibrating system with bi-linear form. Synchronous and subsynchronous fractional components of the response were found. Muszynska and Goldman (1995) presented the analytical, numerical, and experimental simulation of unbalanced rotor/bearing/stator systems with joint looseness or rubbing. The vibrational behavior of such systems
is characterized by orderly harmonic and subharmonic responses, as well as by chaotic patterns of vibrations. The main frequency responses were usually accompanied by a spectrum of higher harmonics. Vibration characteristics of a rotor-bearing system with pedestal looseness were investigated by Chu and Tang (2001). A non-linear mathematical model containing stiffness and damping forces with tri-linear forms was considered. The shooting method was used to obtain the periodic solutions of the system. Stability of these periodic solutions was analyzed by using the Floquet theory. Period-doubling bifurcation and Naimark-Sacker bifurcation were found. Finally, the governing equations were integrated using the fourth order Runge-Kutta method. Different forms of periodic, quasi-periodic and chaotic vibrations are observed by taking the rotating speed and imbalance as the control parameter. Three kinds of routes to or out of chaos, that is, period-to-chaos, quasi-periodic route and intermittence, were found.

A genetic algorithm based inverse problem approach was proposed by He et al. (2003) for the identification of pedestal looseness in the rotor-bearing system. The proposed approach considered the pedestal looseness identification as an inverse problem, and formulated this problem as a multi-parameter optimization problem by establishing a non-linear dynamics model of the rotor-bearing system with the pedestal looseness, and then utilizes a genetic algorithm to search for the solution. In addition, the non-linear dynamics model and the parameter sensitiveness of the system response were investigated. The responses of the system were obtained by using the fourth-order Runge-Kutta integration. The numerical experiments suggest that good identification of the pedestal looseness is possible and the proposed approach is feasible.

The complicated nonlinear phenomena of rotor system with pedestal looseness were analyzed by applying rotor dynamics and nonlinear dynamics theory by Li et al. (2005). By the bifurcation diagrams of the change of rotating speed, it was discovered that vibration of rotor system with pedestal looseness was violent in the sub-critical whirling speed. On the contrary, in the supercritical whirling speed, vibration of rotor was weak. But, under certain conditions, subharmonic resonance could occur and induce violent vibration. In addition, characteristics of vibration of the rotor with pedestal looseness were studied by frequency spectrums. The analysis result provided a theoretical reference for rotor fault diagnosis for rotating machinery.

Because of wrong setting or long-term running of rotating machinery, the looseness may occur in the bearing seats or bases. And also bring impact and rubbing of rotor-stator, that is the looseness and rub-impact coupling fault. A mechanics model and a finite element model of a vertical dual-disk cantilever rotor-bearing system with coupling faults of looseness and rub-impact were set up by Lu et al. (2007). Based on the nonlinear finite element method and contact theory, the dynamical characteristics of the system under the influence of the looseness rigidity and impact-rub clearance
was studied. The results showed that the impact-rub of rotor-stator could reduce the low frequency vibration caused by looseness, and the impact-rub caused by looseness had obvious orientation. Also, the conclusion of diagnosing the looseness and rub-impact coupling faults was given in the end of the paper.

17.6 Shaft Cracks
The presence of various flaws (such as cracks, notches, slits etc.) in any structures and machineries may lead to catastrophic failures. They are particularly likely to occur in instances where shaft stresses are high and where machine has endured many operation cycles throughout its life, so that material failure has occurred as a consequence of fatigue. If the shaft cracks can be detected before catastrophic failure occurs then the machine can be temporarily taken out of service and repaired before situation gets out of hand. The presence of a transverse shaft crack sometimes is detected by monitoring changes in vibration characteristics of the machine. The shaft stiffness at the location of the crack is reduced, by an amount depending on the crack size. This in turn affects the machine natural frequencies, so that changes in natural frequencies may be symptomatic of a shaft crack. Therefore, it demands the detection and the diagnostic (i.e. its localization and sizing) of such flaws so that corrective action can be taken well before it grows critical. Such detection and diagnostic techniques should be practicable in terms of taking experimental measurements. But unfortunately the changes in natural frequency may not occur until the crack has reached a dangerously large size. For this reason most users of rotating machinery depend upon changes in vibration amplitudes, phase and frequency spectrum to detect shaft cracks, rather than on changes in natural frequency.

A transverse crack results in significant changes in both $1\times$ and $2\times$ rotational speed vibration components. The $1\times$ rotational speed component may change in both amplitude (which may either grow or decay) and phase, as a consequence of the change in rotor bending stiffness. A transverse crack also results in a bigger rotor bow due to a steady load (for example gravity) which may be detected under ‘slow roll’ conditions. The classical symptom of a cracked shaft is the occurrence of a vibration component at $2\times$ shaft rotational frequency as a consequence of the shaft asymmetry at the crack location in the presence of a steady load. The $2\times$ rotational frequency component is usually prevalent when the machine is running at a critical speed or $\frac{1}{2}\times$ any critical speed (similar to sub-critical due to gravity). Other types of crack, in particular those which originate from the center of the shaft, may not cause a change in shaft bending stiffness which is significant enough to enable the crack to be detected by $2\times$ rotational speed components in the vibration spectrum.

There are plenty of literatures that deal with the crack (or the flaw) modeling, free and forced vibrations analysis of cracked beams and detection, localization and severity estimation of cracks. Wauer (1990) presented a review of literatures in the field of dynamics of cracked rotors, including
the modeling of the cracked part of structures and determination of different detection procedures to diagnose fracture damages. The review formed a basis for analysing dynamics of cracked beams and columns (i.e. non-rotating, cracked structural elements which is relevant to the cracked rotor problems). Gasch (1993) provided a survey of the stability behaviour of a rotating shaft with a crack and of forced vibrations due to imbalances. Dimarogonas (1996) reviewed the analytical, numerical and experimental investigations on the detection of structural flaws based on changes in dynamic characteristics. Salawu (1997) reviewed the use of natural frequencies as a diagnostic parameter in the structural assessment procedures using the vibration monitoring. The relationship between frequency changes and structural damages were discussed. Various methods proposed for detecting damages by using natural frequencies were reviewed. Factors (e.g. choice of measuring points, effects of ambient conditions on dynamic responses and consistency and reliability of testing procedures, etc.) that could limit successful application of the vibration monitoring for the damage detection and structural assessment were also discussed.

Plethora of crack detection and diagnostic methods is available in literatures based on feature extractions of the free and forced responses, which becomes very complicated and difficult to use in practice. Doebling et al. (1998) provided an overview of methods to detect, locate, and characterize damages in the structural and mechanical systems by examining changes in measured vibration responses. The scope of this paper was limited to methods that use changes in modal properties (i.e. modal frequencies, modal damping ratios, and mode shapes) to infer changes in mechanical properties. The review included both methods that were based solely on changes in the measured data as well as those methods that used the finite element model (FEM) in the formulation. Doebling et al. (1998) classified the vibration based methods into various categories. The methods are broadly based on the linear and nonlinear effects of damage of structures. Another classification system given by Rytter (1993) for damage-identification methods defined four levels of damage identification, as follows: (i) Level 1: Determination of the presence of damage in the structure (ii) Level 2: Level 1 plus determination of the geometric location of the damage (iii) Level 3: Level 2 plus quantification of the severity of the damage and (iv) Level 4: Level 3 plus prediction of the remaining service life of the structure. Vibration-based damage identification methods that do not make use of some structural model primarily provide Level 1 and Level 2 damage identification. When vibration-based methods were coupled with a structural model, Level 3 damage identification could be obtained in some cases. Level 4 prediction was generally associated with the fields of fracture mechanics, fatigue-life analysis, or structural design assessment.

Sabnavis et al. (2004) presented a review of literatures published since 1990 and some classical papers on the crack detection and severity estimation in shafts. The review was based on three
categories namely vibration-based methods, modal testing and non-traditional methods. They also discussed the types and causes of rotor cracks and fundamentals of the crack propagation.

In addition to aforementioned reviews, several authors contributed to the development of ‘cracked element’, analysis of cracked beams and crack detection using changes in natural frequencies. Dimarogonas and Paipetis (1983) showed a beam with a transverse crack, in general, can be modeled in the vicinity of the crack by way of a local flexibility (compliance) matrix, connecting the longitudinal force, bending moment and shear force and corresponding displacements. If torsion is also added, a 6×6 compliance matrix may result and it has off-diagonal terms that indicate coupling of the respective forces and displacements, and therefore coupling of respective motions. This matrix is a diagonal matrix in the absence of crack for the symmetrical beam cross-section. Gounaris and Dimarogonas (1988) developed the Euler–Bernoulli beam cracked element based on the fracture mechanics approach. Theoretically, coefficients of the compliance matrix are computed based on available expressions of the stress intensity factor (SIF) and associated expressions of the strain energy density function (SEDF) by using the linear fracture mechanics approach. Fernandez-saez et al. (1999) obtained an expression for the closed form fundamental natural frequency of a cracked Euler-Bernoulli beam by applying the Rayleigh method. This approach was applied to simply supported beams with a rectangular cross-section having a crack in any location of the beam span. Behzad and Bastami (2004) investigated the effect of axial forces on changes in natural frequencies of shafts.

Lee et al. (1992) proposed a switching crack model with two different stiffness states, depending upon whether the crack is open or close. The necessary conditions for the crack opening and closing were analytically derived from the simple rotor with the switching crack.

Darpe et al. (2003a) analyzed the coupling of the lateral and longitudinal vibrations due to the presence of transverse surface crack in a rotor for the non-rotating and rotating conditions. The steady state unbalance response of a cracked rotor with a single centrally situated crack subjected to periodic axial impulses was investigated experimentally. Darpe et al. (2003b) studied a simple Jeffcott rotor with two transverse surface cracks and the effect of the interaction of two cracks on the breathing behavior and on the unbalance response of the rotor. They noticed the significant changes in the dynamic response of the rotor when the angular orientation of one crack relative to the other was varied. Darpe et al. (2004) studied the coupling between longitudinal, lateral and torsional vibrations together for a rotating cracked shaft with a response-dependent non-linear breathing crack model. Crack signatures were obtained by using external excitations and the excitation in one mode led to an interaction between all the modes, as the couplings were accounted. The co-existence of frequencies of other modes in the frequency spectra of a particular mode and the presence of sum and difference
of frequencies around the excitation frequencies and its harmonics were used as the indicators for crack diagnosis. Darpe et al. (2006) formulated equations of motion of the rotor with a transverse surface crack with a bow, and analyzed the steady state and the transient response of the rotor. They assessed the effect of the residual bow on the stiffness characteristic of the rotating cracked shaft and observed that the usual level of bow might not significantly influence the stiffness variation and the nonlinear nature of crack response was not significantly altered.

Chondros (2005) developed a variational formulation for the torsional vibration of a cylindrical shaft with a circumferential crack. The Hu-Washizu-Barr (1955, 1966) variational formulation was used to develop the differential equation and boundary conditions of the cracked rod. He reported independent evaluations of crack identification methods in rotating shafts and compared with methods using the continuous crack flexibility theory.

Di and Law (2006) provided different types of damage models of a frame element, whose elemental matrices were decomposed into their eigen value and eigen vector matrices. These eigen-parameters were included in a flexibility-based and sensitivity-based model-updating algorithm for the condition assessment of a plane frame structure. Lei et al. (2007) proposed a finite element model for vibration analysis of a crankshaft with a slant crack in the crankpin and investigated the influence of the crack depth on the transient response of a cracked crankshaft.

Lee and Chung (2000) presented a nondestructive evaluation procedure for identifying the crack (i.e. the location and the size of the crack), in a one-dimensional beam-type structure using the natural frequency data. Lowest four natural frequencies were obtained of the cracked structure by FEM and the approximate crack location was obtained by Armon's Rank-ordering method (1994). The actual crack location was identified by Gudmundson's equation (1982) using the determined crack size and aforementioned natural frequencies. Morassi (2001) dealt with the identification of a single crack in a vibrating bar based on the knowledge of the damage-induced shifts in a pair of natural frequencies. The crack was simulated by an equivalent linear spring, which was connected between two segments of the bar. The analysis was based on an explicit expression of the frequency sensitivity to damage and enables non-uniform bars under general boundary conditions to be considered. Owolabi et al. (2003) developed a method to detect the presence of a crack in beams and determine its location and size, based on experimental modal testing methods. Changes in natural frequencies and frequency response function amplitudes as a function of crack depths and its locations were used in the crack detection methodology. Most of the literatures discussed on crack identification methods are based on free vibrations.
Most of the model based crack detection and diagnostics are based on the procedure that the experimental measurements from prototype structures are compared with predicted measurements from a corresponding finite element model. One way to compare the data is to reduce the number of degrees of freedom in the analytical model. This is generally achieved by implementing the condensation scheme. Guyan (1965) introduced a static reduction method, based on the assumption that inertia terms are negligible at low frequency of excitations, in order to reduce mass and stiffness matrices by eliminating dofs corresponding to slave nodes (e.g. where no force is applied). Hence, any frequency response functions generated using the reduced equation of motion is exact only at zero frequency. As the excitation frequency increases, the inertia terms neglected become more significant.

Dharmaraju et al. (2004, 2005) and Tiwari and Dharmaraju (2006) developed algorithms for identifying the crack flexibility coefficients and subsequently estimation of the equivalent crack depth based on the forced response information. They outlined the condensation scheme for eliminating the rotational degree of freedoms at crack element nodes based on the physical consideration of the problem, which was otherwise difficult to eliminate. However, the main practical limitation of the algorithm was that the location of the crack must be known a priori. Also the algorithm used the Euler-Bernoulli beam theory in the beam model without considering the damping in the system. Recently, Karthikeyan et al. (2007a and b) developed an algorithm for the crack localization and the sizing in a cracked beam based on the free and forced response measurements without applying condensation schemes, which was the main practical limitation in application of the work. Moreover, the method was also based on the mode shape measurement, which is relatively difficult to measure. The hybrid reduction scheme was extended by Karthikeyan et al. (2008), by including the damping, to the crack localization and sizing algorithm based on purely the forced response measurements, which is a more controlled and accurate way of measurements. The main contribution of the paper was the elimination of measurement of rotational dofs completely, which otherwise difficult to measure accurately. The application of the regularization technique helped in estimating both the crack size and its location, iteratively, without which the algorithm might lead to unbounded estimates of crack parameters. The converged value of the crack depth ratio and corresponding crack location, up to a desired accuracy, is considered as the final size and the location of the actual crack in the cracked beam. For illustrations, beams with the simply support and cantilever end conditions have been considered. The convergence of the algorithm has been found to be very fast and the robustness of the algorithm was tested by adding the measurement error in the resonant frequency measurement and the measurement noise in force.

### 17.7 Rolling Element Bearing Faults

The primary example of characteristics associated with a specific component is the frequencies generated by defects and flaws in rolling element bearings. Faults in rolling bearings may occur prematurely as a consequence of operating the bearing under in appropriate loading conditions.
(including misalignment) and at excessive speeds. Alternatively they may be produced simply as part of the normal wear process during the life of the bearing. Traditionally machines with rolling element bearings would have their bearings renewed regularly, as part of the normal maintenance schedule, irrespective of their wear. This would be done in an effort to avoid a bearing failure at a later time, which would necessitate machine stoppage, at an inconvenience (and more costly) moment. The growing trend, however, is to monitor the condition of rolling element bearings continually so that bearing wear may be detected at an early stage, and enables the engineer to ensure that the bearing is replaced at a convenient time before the bearing fails completely. In this way bearings are replaced only when they are worn out, not as a matter of routine, and at the same time the machine overhaul can be well ordered and planned in advance. Condition monitoring of rolling element bearings has enabled cost saving of over 50%; as compared with traditional maintenance method. The most common method of monitoring the condition of rolling element bearings is to measure the vibration of the machine at the baring at regular intervals using a velocity transducer or an accelerometer mounted on the machine casing. More recently, observation of the bearing outer-race deformations using fiber optics and high sensitivity proximity transducers has also been used to monitor bearing condition. Defects in the bearing may develop on either raceway, on the rolling elements themselves, or on the cage; subsequent vibrations are forced as a consequence of impact between the fault and other bearing components, so that the frequency of the resulting vibrations is largely dependent on the frequency of impacting. For example, a defect in a bearing outer raceway would set up vibrations corresponding to the frequency with which rolling elements passed over the defects. Consideration of the bearing kinematics (geometry as shown in Figure 17.11) enables calculation of the frequencies associated with defects in different bearing components.

![Figure 17.11 Rolling element bearing macro-geometry](image-url)
Outer race frequency = \( \frac{n N}{2 \times 60} \left( 1 - \frac{d}{D} \cos \alpha \right) \) \hspace{1cm} (11.9)

Inner race frequency = \( \frac{n N}{2 \times 60} \left( 1 + \frac{d}{D} \cos \alpha \right) \) \hspace{1cm} (11.10)

Rolling element frequency = \( \frac{D N}{60} \left[ 1 - \left( \frac{d}{D} \right)^2 \cos^2 \alpha \right] \) \hspace{1cm} (11.11)

Case frequency = \( \frac{N}{120} \left[ 1 - \frac{d}{D} \cos \alpha \right] \) \hspace{1cm} (11.12)

where \( N \) is the shaft frequency in rev/min, \( \alpha \) is the contact angle, and \( n \) is the number of rolling elements.

Whilst the characteristic frequencies can be easily calculated, the process of diagnosing a fault can be complicated by a number of factors. Some of the characteristics frequencies may be very close to harmonics of rotational speed; this means that a narrow-band spectrum analyzer is required in order to distinguish vibration components caused by a bearing failure those caused by for example, unbalance. Even when the instrumentation provides a suitable resolution in the frequency domain, additional ‘sum and difference’ frequencies may be apparent as a result of interaction between two or more characteristic frequencies. Such sum and difference frequencies appear as sidebands i.e. frequency peaks either side of the higher main frequency component of the signal. As wear of the bearing progresses the frequency spectrum changes further. Sometimes higher-order harmonics of the defect frequency become present, sometimes with their own sidebands, and can dominate the spectrum. In addition, wear particles are transported around the bearing and accelerate the development of further defects at other locations, leading to high level of vibration at many frequencies so that peaks which are characteristic of particular defect difficult to distinguish. A most important feature of condition monitoring of bearings (and rotating machinery in general), is the collection of ‘baseline’ reference measurements of vibration taken when the machine is first commissioned (or re-commissioned after overhaul). It is only when the engineer is in possession of these that a confident diagnosis of a significance and cause of peaks in the vibration spectrum can be made. Micro-irregularity on bearing contacting surface gives very high frequency components in the response.

An alternative method of monitoring rolling element bearing condition is the ‘shock pulse method’ (SPM). This method depends upon the signal input to the measuring instrument being passed through...
a narrow-band filter whose center frequency corresponds to the accelerometer natural frequency. As part of the bearing collides with a defect, a shock wave is transmitted through the bearing and machine casting thus exciting the accelerometer by means of a pulse input. The accelerometer output is a damped transient waveform whose frequency is much higher than the frequencies characteristic of specific bearing faults, and whose magnitude is dependent upon the magnitude of the defect in the bearing. Again this method relies heavily upon establishing reliable baseline data and on monitoring changes in output level rather than on absolute values. SPM cannot indicate the cause of the vibration levels, but certainly it is a less expensive technique.

In acoustic flaw detection (incipient flaw detection or IFD) the band from 80 kHz to 120 kHz is responsive to bearing defects and being used in most operational applications. This choice is highly empirical, however, and other bands are being used. Following preamplification and bandpass filtering to eliminate as much noise and extraneous information as possible, the acoustic flaw detection signal is conditioned to produce three outputs which can be measured as numerical values on a meter. First, the RMS amplitude within the bandpass is determined and displayed as a measure of overall condition. Next, the energy content of the spikes or pulses in the high frequency signal above some arbitrary and automatically set multiple, usually 2-4 times the RMS level, is detected and displayed as indicator of discrete flaws. This measurement can be thought of as the energy content above some fixed crest factor. Finally, the rate at which the signal crosses a given threshold is counted and displayed. Because large signals will produce more crossings per event or pulse, the count rate measurement can be a second a second approximation of the energy or severity of the defect. Acoustic flaw detection thus provides three measurements with which to judge overall condition as well as permits recognition and evolution of local defects. As indicators of bearing condition, field experience ranks the RMS (root mean square) measurement first, followed by SAT with count rate a distant third.

A review of vibration and acoustic measurement methods for the detection of defects in rolling element bearings was presented by Tandon and Choudhury (1999). Detection of both localized and distributed categories of defect was considered. An explanation for the vibration and noise generation in bearings was given. Vibration measurement in both time and frequency domains along with signal processing techniques such as the high-frequency resonance technique were covered. Other acoustic measurement techniques such as sound pressure, sound intensity and acoustic emission were reviewed. Recent trends in research on the detection of defects in bearings, such as the wavelet transform method and automated data processing, were also been included.

While operating a rolling bearing with local faults impulse is created, the high-frequency shock vibration is then generated and the amplitude of vibration is modulated by the pulse force. The envelope analysis method provides an important and effective approach to analyse the fault signals of
high-frequency impact vibration, it has been applied to the fault diagnosis of rolling bearings successfully (Radcliff, 1990; Randall, 1986; Brown, 1989). However, in the traditional envelope analysis method, the fault is identified through the peak value of envelope spectrum. Thus, this traditional method has two aspects of disadvantages. On the one hand, FFT method is widely used in the spectrum analysis of envelope signals; however, it could only give the global energy-frequency distributions and fail to reflect the details of a signal. So it is hard to analyse a signal effectively when the fault signal is weaker than the interfering signal (Ho and Rand, 2000; Randall, 1997; McFadden, Smith, 1984). At the same time, it is easy to diffuse and truncate the signal’s energy as FFT regards harmonic signals as basic components, which will lead to energy leakage and cause lower accuracy. On the other hand, the central frequency of the filter is determined with experience while forming an envelope signal, which will make great subjective influence on the results (McFadden, Smith, 1984; Tse, et al., 2001; Randall and Gao, 1996).

Generally speaking, the process of the roller bearing fault diagnosis consists of three steps: (1) the collection of the roller bearing fault vibration signals; (2) the extraction of the fault features; (3) condition identification and fault diagnosis. How to extract the fault features and identify the condition from the roller bearing vibration signals are the key steps in the fault diagnosis of roller bearings. As the fault vibration signals of roller bearings are non-stationary, it is a crux how to obtain feature vectors from them for the fault diagnosis. The traditional diagnosis techniques perform this from the waveforms of the fault vibration signals in the time or frequency domain, and then construct the criterion functions to identify the working condition of roller bearings. However, because the non-linear factors such as loads, clearance, friction, stiffness and so on have distinct influence on the vibration signals due to the complexity of the construct and working condition of roller bearings, it is very difficult to make an accurate evaluation on the working condition of roller bearings only through the analysis in time or frequency domain (Sandy, 1988; Li and Wu, 1989). The main purpose of this paper is to propose a new fault feature extraction method for roller bearing fault diagnosis. Features are those parameters derived from the measured data that robustly indicate the presence of the roller bearing faults. The main feature extraction methods include: time-domain methods, frequency-domain methods, and time–frequency methods. Time domain methods such as peak amplitude, root-mean-square amplitude, crest factor, kurtosis and shock pulse counting have been successfully applied to roller bearings (Ma and Li, 1993; Martin, 1989; Volker and Martin, 1986). Frequency domain methods applied to roller bearing fault diagnosis include Fourier spectra time waveform, cepstrum analysis, sum and difference frequencies analysis and the envelope spectra technique (Ma and Li, 1993; Radcliff, et al., 1990; Peter, 2001; Zheng and Wang, 2001). A comparative study of various feature extraction methods that fall into the time-domain and frequency-domain methods is presented in (Elbestawi, and Tait, 1986). As the roller bearing vibration signals possess non-stationary characteristic, time–frequency methods is effective to extract the feature of the original data. The
wavelet transform has been applied to feature extraction for roller bearing vibration signals and efficient results have been obtained (Li and Ma, 1997; Geng and Qu, 1994; Lin and Qu, 2000). The mathematical model need to be established or the fault mechanism of the roller bearing vibration system need to be studied before the feature extraction in above-mentioned methods. For example, in the envelope spectra technique, the centre frequency and bandwidth of the band-pass filter must be determined correctly while forming envelope signal and the fault characteristic frequency of the roller bearing must be computed (Randal, 1986; Randal and Gao, 1996). However, for a complex roller bearing vibration system, the related parameters and the mathematical model are difficult to be determined. In many cases, these parameters (such as the centre frequency of the band-pass filter in envelope spectra technique) are determined with experience, which will make great subjective influence on results.

The application of wavelets has emerged in the context of damage detection, and an excellent review of this is given in (Staszewski, 1998). Mori et al. (1996) have predicted the spalling on the ball bearing by applying discrete wavelet transform to vibration signals. Jing and Qu (2000) have proposed a denoising method based on Morlet wavelets for feature extraction and they have successfully applied it to inner race fault detection of the roller bearing. However, the previous works (Jing and Qu, 2000; Mori et al., 1996; Li and Jun, 1992) dealt with the detection of one fault in a bearing using wavelet transform. In the present study, the diagnosis of single and multiple ball bearing race faults has been investigated using discrete wavelet transform. Bearing race faults were detected by using discrete wavelet transform (DWT) by Prabhakar et al. (2002). Vibration signals from ball bearings having single and multiple point defects on inner race, outer race and the combination faults were considered for analysis. The impulses in vibration signals due to bearing faults were prominent in wavelet decompositions. It was found that the impulses appear periodically with a time period corresponding to characteristic defect frequencies. It was shown that DWT could be used as an effective tool for detecting single and multiple faults in the ball bearings.

**Example 17.2** The physicals and calculated frequencies for the Machine Faults Simulator (MFS) pillow block bearings (MB ER-10K) are as follows: (i) quantity of rolling elements $Z$: eight (8) balls, (ii) ball diameter ($D_b$): 0.3125 inch, (iii) pitch diameter ($D_m$): 1.319 inch and (iv) contact angle ($\alpha$): 0 degree.

**Solution:** To calculate the BPFO multiplier, the following formula is used:

$$BPFO = \frac{n}{2} \left(1 - \frac{d}{D \cos \theta}\right)$$
To calculate the BPFI multiplier, the following formula is used:

\[ BPFI = \frac{n}{2} \left( 1 + \frac{d}{D} \cos \theta \right) \]

To calculate the BSF multiplier, the following formula is used:

\[ BSF = \frac{D}{2d} \left( 1 - \left( \frac{d}{D} \right)^2 \cos^2 \theta \right) \]

To calculate the FTF multiplier, the following formula is used:

\[ FTF = \frac{1}{2} \left( 1 - \left( \frac{d}{D} \right) \cos \theta \right) \]

On putting the parameters into the above formulas we get the different faults frequency multipliers as listed in Table 17.1.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Rotor Brgs 5/8”</td>
<td>MB</td>
<td>ER-10K</td>
<td>8</td>
<td>0.3125”</td>
<td>1.319”</td>
<td>0.3815</td>
<td>3.052</td>
<td>4.948</td>
<td>1.992</td>
</tr>
</tbody>
</table>

The machine -fault simulator was configured to eliminate all defects, and good bearing was installed in both the bearing housing with and without the bearing loader and baseline data was collected. The bearing loader was installed center hung adjacent to the outboard bearing. Baseline data was collected at several speeds (10Hz, 20Hz, 30Hz).

After that the defective bearing was then installed in the outboard position. Data were collected at similar speeds as previously. Baseline measurements were compared with the vibration signatures of the defective bearings. Tables 17.2-17.4 illustrate the trends of fault frequencies and adjacent harmonic frequencies at different rotational speeds.
Table 17.2 Fault frequencies and adjacent harmonic frequencies for 10Hz shaft speed

<table>
<thead>
<tr>
<th>Notation</th>
<th>Fault Frequency multiplier</th>
<th>Fault frequency (Hz)</th>
<th>Harmonic frequencies (Hz)</th>
</tr>
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<tbody>
<tr>
<td>BPFI</td>
<td>4.948</td>
<td>49.48</td>
<td>50</td>
</tr>
<tr>
<td>BPFO</td>
<td>3.052</td>
<td>30.52</td>
<td>30</td>
</tr>
<tr>
<td>BSF</td>
<td>1.992</td>
<td>19.92</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 17.3 Fault frequencies and adjacent harmonic frequencies for 20Hz shaft speed

<table>
<thead>
<tr>
<th>Notation</th>
<th>Fault Frequency multiplier</th>
<th>Fault frequency (Hz)</th>
<th>Harmonic frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPFI</td>
<td>4.948</td>
<td>98.96</td>
<td>100</td>
</tr>
<tr>
<td>BPFO</td>
<td>3.052</td>
<td>61.04</td>
<td>60</td>
</tr>
<tr>
<td>BSF</td>
<td>1.992</td>
<td>39.84</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 17.4 Fault frequencies and adjacent harmonic frequencies for 30Hz shaft speed

<table>
<thead>
<tr>
<th>Notation</th>
<th>Fault Frequency multiplier</th>
<th>Fault frequency (Hz)</th>
<th>Harmonic frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPFI</td>
<td>4.948</td>
<td>148.44</td>
<td>150</td>
</tr>
<tr>
<td>BPFO</td>
<td>3.052</td>
<td>91.56</td>
<td>90</td>
</tr>
<tr>
<td>BSF</td>
<td>1.992</td>
<td>59.76</td>
<td>60</td>
</tr>
</tbody>
</table>

Figures 17.12-17.15 illustrates of typical frequency spectra showing the inner race fault, outer race fault, rolling element fault and combination fault at the shaft running speed of 10Hz. In these figures the vibration data collected from the sensor was placed at the bearing housing in the vertical direction. In the spectra, the harmonics of running frequency as well as the fault frequency was also appeared.

Figure 17.12 The frequency spectra illustrates the inner race fault frequency (shaft speed of 10Hz)
Figure 17.13 The frequency spectra illustrates outer race fault frequency (shaft speed of 10Hz)

Figure 17.14 Frequency spectra illustrated rolling element fault frequency, running at 10Hz

Figure 17.15 The frequency spectra illustrates combine fault frequency (shaft speed of 10Hz)
17.8 Faults in Gears

Vibration measurements taken at the bearings can also indicate the condition of the gearbox. Gears typically generates a complex, broad vibration spectrum beginning with frequencies well below the shaft rotational speeds and extending to several multiples of the gear mesh frequency (i.e., number of gear teeth times shaft rpm). The amplitude at mesh frequency may vary greatly from gear to gear, depending on number of teeth, gear ratio, tooth surface finish and load. As a general rule, the amplitude at mesh frequency will be smaller with (i) larger number of teeth (ii) lower gear ratio (iii) higher quality of tooth finish and (iv) lower load applied to the gear. A narrow band spectrum analyzer is very useful for this purpose, because the monitoring process involves the detection of discrete frequency components that must be distinguished from frequencies generated through other mechanisms (i.e., rolling element bearings). The characteristic frequency of a particular gear set is the gear mesh frequency (the product of the number of teeth on gear and the shaft rotational frequency), which will be evident in the spectrum relating to any gearbox, in good condition or otherwise. When one gear becomes damaged the gear mesh frequency component of vibration may increase substantially as compared to the base line vibration measurements, but this is not always the case. Harmonics of gear mesh frequency may also become more apparent. Another frequency, which is often excited by gear defects, is the resonant frequency of geared shaft itself. This frequency can usually be measured by impulse testing. Both the natural frequency of the geared shaft and the gear mesh frequency may have accompanying side bands; sometimes side bands themselves may be the main indicator of a defective gearbox (Figs. 17.16 and 17.17).

Figure 17.16 The typical spectrum for a healthy pinion with indication of the mesh frequency and its sidebands at no-load condition (shaft speed of 10Hz)
Recent trends: Gearboxes play an important role in industrial applications. Typical faults of gears include pitting, chipping, and more seriously, crack. When a gear has a local fault, the vibration signal of the gearbox may contain amplitude and phase modulations that are periodic with the rotation frequency of the gear. The modulation of the meshing frequency, as a result of faulty teeth, generates sidebands, which are frequency components equally spaced around a centre frequency. The centre frequency called the carrier frequency may be the gear mesh frequency, multiples of bearing ball pass frequency, resonant frequency of a machine component/structure, or the resonant frequency of an accelerometer. Sidebands are either the shaft rotational speed or one of its multiples. It is well known that the most important components in gear vibration spectra are the tooth-meshing frequency and their harmonics, together with sidebands. Amplitude modulations are present when a gear meshes an eccentric gear or a gear riding on a bent or misaligned shaft. If there is a local gear fault, the gear angular velocity could change as a function of the rotation. As a result of the speed variation, frequency modulations occur. In many cases, both amplitude and frequency modulations are present. The increasing in the number and the amplitude of such sidebands often indicates faulty conditions (Dalpiaz, 2000). Since modulating frequencies are caused by certain faults of machine components including gear, bearing, and shaft, the detection of the modulating signal is very useful to detect gearbox fault. In the early stage of a fault, fault symptoms are not obvious. Due to unsteady shaft/gear rotating speed, lubrication situation, tooth stiffness variations, and other reasons, the collected vibration signal from a gearbox is usually non-stationary. Sideband distance may vary with time. As a
result, the spectral composition of the collected vibration signal often changes with time. Fourier analysis is unable to reveal such characteristics (Yesilurt, 2000). Other approaches are needed to identify early fault features from non-stationary signals.

The process of restoring the modulating signal that is mixed with a carrier signal is called demodulation. The detection of the modulating signal is traditionally realised through identification of sidebands in the frequency domain. However, when a signal acquired is transient in nature, spectral analysis is inherently unsuitable for detection of sidebands (Ma and Li, 1996). Hilbert transform (HT) has been shown to be useful for demodulation (Feldman, 1997). However, it is unable to show frequencies clear enough for visual inspection. Therefore, by itself, it cannot reveal early fault signatures buried in non-stationary signals. Time–frequency analysis offers an alternative method to signal analysis by presenting information in the time–frequency domain simultaneously. The short-time Fourier transform is probably the most widely used time–frequency representation. However, its resolution is often unsatisfactory because of poorly matched windows in signal analysis (Jones and Parks, 1992). Though Wigner–Ville distribution, another time–frequency method, is also used in fault detection (McFadden and Wang, 1992), the oscillating interference between the signal components often exists when applied to a multi-component signal (Yesilurt, 2000). The Choi–Williams distribution provides better resolution than the smoothed Wigner–Ville distribution; however, it is still insensitive to the time-scale of signal components (Jones and Parks, 1992). The smoothed instantaneous power spectrum method, which combines the advantages of the spectrogram and the instantaneous power spectrum, gives clearer time–frequency representation of signal (Yesilurt, 2000). However, weak signals may be lost in the process. (Fan and Zuo, 2006) proposed a fault detection method that combines Hilbert transform and wavelet packet transform. Both simulated signals and real vibration signals collected from a gearbox dynamics simulator were used to verify the method. Analysed results showed that the proposed method was effective to extract modulating signal and help to detect the early gear fault.

Condition monitoring of machines is gaining importance in industry because of the need to increase reliability and to decrease possible loss of production due to machine breakdown. The use of vibration and acoustic emission (AE) signals is quite common in the field of condition monitoring of rotating machinery. By comparing the signals of a machine running in normal and faulty conditions, detection of faults like mass unbalance, rotor rub, shaft misalignment, gear failures and bearing defects is possible. These signals can also be used to detect the incipient failures of the machine components, through the on-line monitoring system, reducing the possibility of catastrophic damage and the down time. Some of the recent works in the area are listed in (Shiroishi et al., 1997; McFadden, 2000; Nandi, 2000; Randall, 2001; Al-Balushi, and Samanta, 2002; Antoni and Randall, 2002; McCormick and Nandi, 1997; Dellomo, 1999). Although often the visual inspection of the frequency domain
features of the measured signals is adequate to identify the faults, there is a need for a reliable, fast and automated procedure of diagnostics.

Artificial neural networks (ANNs) have been applied in automated detection and diagnosis of machine conditions (Nandi, 2000; McCormick and Nandi, 1997; Dellomo, 1999; Samanta, and Al-Balushi, 2001 and 2002) treating these as classification or generalisation problems based on learning pattern from examples or empirical data modelling. However, the traditional neural network approaches have limitations on generalisation giving rise to models that can overfit to the training data. This deficiency is due to the optimisation algorithms used in ANNs for selection of parameters and the statistical measures used to select the model. Recently, support vector machines (SVMs), based on statistical learning theory, are gaining applications in the areas of machine learning, computer vision and pattern recognition because of high accuracy and good generalisation capability (Burges, 1998; Guyon and Christianini, 1999; Scholkopf, 1998; Gunn, 1998; Vapnik, 1999). The main difference between ANNs and SVMs is in the principle of risk minimisation (RM) (Gunn, 1998). In case of SVMs, structural risk minimisation (SRM) principle is used minimising an upper bound on the expected risk whereas in ANNs, traditional empirical risk minimisation (ERM) is used minimising the error on the training data. The difference in RM leads to better generalisation performance for SVMs than ANNs. The possibilities of using SVMs in machine condition monitoring applications are being considered only recently (Nandi, 2000; Jack and Nandi, 2000). In (Jack and Nandi, 2000), a procedure was presented for condition monitoring of rolling element bearings comparing the performance of two classifiers, ANNs and SVMs, with all calculated signal features and fixed parameters for the classifiers. In this, vibration signals were acquired under different operating speeds and bearing conditions. The spectral data and the statistical features of the signals, both original and with some preprocessing like differentiation and integration, low- and high-pass filtering were used for classification of bearing conditions. However, there is a need to make the classification process faster and accurate using the minimum number of features which primarily characterise the system conditions with an optimised structure of ANNs and SVMs (Nandi, 2000; Jack and Nandi, 2000). Genetic algorithms (GAs) were used for automatic feature selection in machine condition monitoring (Nandi, 2000; Jack and Nandi, 2002; Samanta et al., 2001). In (Jack and Nandi, 2002), the procedure of Jack and Nandi (2000) was extended to introduce a GA-based approach for selection of input features and classifier parameters, like the number of neurons in the hidden layer in case of ANNs and the radial basis function (RBF) kernel parameter, width, in case of SVMs. The features were extracted from the entire signal under each condition and operating speed. In Samanta et al. (2001), some preliminary results of ANNs and GAs were presented for fault detection of gears using only the time-domain features of vibration signals. In this approach, the features were extracted from finite segments of two signals: one with normal condition and the other with defective gears.
Final Remarks

To summarise, in the present chapter based on the vibration measurement the condition monitoring of rotating machinery has been described. Basic frequency components which may appear in the vibration signal due to various fault conditions have been described. Some basic fault conditions like the unbalances, misalignments, cracks, rubs, loose components, etc. have been discussed in detailed. Some basic rotating machine element fault conditions like gears, bearings, coupling, etc. have been described. The method presented in the chapter for detecting the fault condition is very primitive; however, easy and effective in most of the practical cases, however, it involves human (expert) judgments. The subsequent chapter, deals with the active control of rotor by magnetic bearing by monitoring the vibration of the rotor, which is the main machine element gives dynamic forces to the support structures.

Exercise Problem

Exercise 17.1 A limestone crusher unit has a three-stage gearbox with the total reduction of speed of 1:33.5. The gearbox is driven by a 300 kW electric motor with the input speed of 1493 rpm. The number of teeth of various stages of the pinion are 31, 21, and 25; and that of gears are 66, 87, and 95. Find the possible gearmesh frequencies, which might be present in the vibration spectrum measured from the gearbox housing. Find the sampling interval you would like to choose to measure the vibration signal from the gearbox to detect all the gearmesh frequencies correctly.

Exercise 17.2 Choose a single answer from multiple choice answers

(i) The plot between the vibration amplitude versus frequency and the phase versus frequency is called

(A) Campbell diagram (B) Nyquist plot (C) Bode plot (D) water-fall diagram
Appendix: 17.A

17.A Discrete wavelet transform

Wavelets provide a time-scale information of a signal, enabling the extraction of features that vary in time. This property makes wavelets an ideal tool for analyzing signals of a transient or non-stationary nature. The continuous wavelet transform (CWT) of \( f(t) \) is a time-scale method of signal processing that can be defined as the sum over all time of the signal multiplied by scaled, shifted versions of the wavelet function \( \Psi(t) \). Mathematically,

\[
\text{CWT}(a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \Psi^*(\frac{t-b}{a}) \, dt
\]

(17.A1)

where \( \Psi(t) \) denotes the mother wavelet. The parameter \( a \) represents the scale index which is a reciprocal of frequency. The parameter \( b \) indicates the time shifting (or translation). The discrete wavelet transform (DWT) is derived from the discretization of CWT \((a, b)\) and the most common discretization is dyadic, given by

\[
\text{DWT}(j, k) = \frac{1}{\sqrt{2^j}} \int_{-\infty}^{\infty} f(t) \Psi^*\left(\frac{t-2^j k}{2^j}\right) \, dt
\]

(17.A2)

where \( a \) and \( b \) are replaced by \( 2^j \) and \( 2^k \). An efficient way to implement this scheme using filters was developed in 1989 by Mallat (1989). The original signal, \( f(t) \), passes through two complementary filters and emerges as low frequency [approximations (A’s)] and high frequency [details (D’s)] signals. The decomposition process can be iterated, with successive approximations being decomposed in turn, so that a signal can be broken down into many lower-resolution components.
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